PBAR Note 653
Quadrupole Focussing and Quadrupole Steering Tolerances for the 8 GeV AP3-P1 Lattice

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INTRODUCTION

This note will outline the sensitivity to emittance blowup of antiproton transfers due to quadrupole focussing and quadrupole steering errors in the 8 GeV AP3-P1 transfer line. It will be shown that these tolerances are much larger than the dipole steering tolerances discussed in PBAR Note 649. This note will use the lattice discussed in PBAR Note 648.

QUADRUPOLE FOCUSSING ERRORS

The emittance blowup due to quadrupole focussing errors is discussed in PBAR Note 639. (There is a correction to this note outlined in PBAR Note 647.) The change in emittance due to focussing and steering errors two synchrotrons connected via a beam-line is:

$$\varepsilon_2 = \frac{1}{2}\left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} + \frac{(\alpha_2\beta_1 - \alpha_1\beta_2)^2}{\beta_1\beta_2}\right]\varepsilon_1 + 3\left[\beta_2(\Delta\theta)^2 + 2\alpha_2(\Delta\theta)(\Delta x) + \gamma_2(\Delta x)^2\right] + 3\left[\beta_2(\Delta D')^2 + 2\alpha_2(\Delta D')(\Delta D) + \gamma_2(\Delta D)^2\right] \times \left(\frac{\sigma_p^2 + (\Delta p)^2}{p_2^2}\right) \quad (1)$$

The subscripts 1 and 2 respectively denote the beam-line and downstream storage ring parameters. All (ΔK) quantities are absolute differences: $|K_1 - K_2|$. The three terms are respectively TWISS parameter, injection steering and dispersion mismatches. If the ΔX quantities are not treated as absolute differences, then the cross terms for the injection steering and dispersion could lessen the blowup depending up the sign of ($K_1 - K_2$). (PBAR Note 647).

Figure 1 and Table 1 show the amount of current change from the nominal needed for a 10% emittance blowup ($\varepsilon_2 = 1.10\ \varepsilon_1$) with ε_1=2.0 π–mm-mrad (95% unnormalized) and σ_p/p = 0.00075. It was assumed that $\Delta\theta$, Δx, and Δp=0. It was also assumed at nominal currents that the beam line is matched to the Main Injector.

QUADRUPOLE STEERING ERRORS

The amount of error kick generated by a quadrupole from the nominal trajectory is:

$$|\Delta\theta| = |\Delta k L u| \quad (2)$$

where $\mathbf{\Delta k}$ is the error in the gradient, **L** is the length of the quadrupole, and **u** (horizontal or vertical) is the nominal trajectory of the beam through the quadrupole measured with respect to the magnetic center of the quadrupole. The quadrupole gradient is defined as:

$$k = \frac{1}{B\rho}\frac{dB_y}{dx} \tag{3}$$

The gradient is assumed to be proportional to the excitation current

$$\frac{\Delta k}{k} = \frac{\Delta I}{I} \tag{4}$$

The magnitude of the betatron amplitude error is:

$$|\Delta A| = \sqrt{\beta_Q}\,|\Delta\theta| \tag{5}$$

where β_Q is the lattice function at the quadrupole. The emittance growth (95% normalized) due to the betatron amplitude error is:

$$\Delta\varepsilon_n = 3\gamma_r\beta_r|\Delta A|^2 \tag{6}$$

where $\gamma_r\beta_r$ are the Lorentz relativistic parameters. The amount of current needed to cause a given emittance growth is:

$$|\Delta I| = \frac{I}{\sqrt{\beta_Q}\,|\Delta kLu|}\sqrt{\frac{\Delta\varepsilon_n}{3\gamma_r\beta_r}} \tag{7}$$

Figure 2 and Table 2 give the amount of current error needed to cause a 1.0 π-mm-mrad emittance blowup (95% normalized) with a 1 mm nominal trajectory at a quadrupole. Equation 7 shows that the current tolerance is inversely proportional to the trajectory offset. The lattice functions at the quadrupoles from PBAR Note 648 are tabulated in Tables 3 and 4.

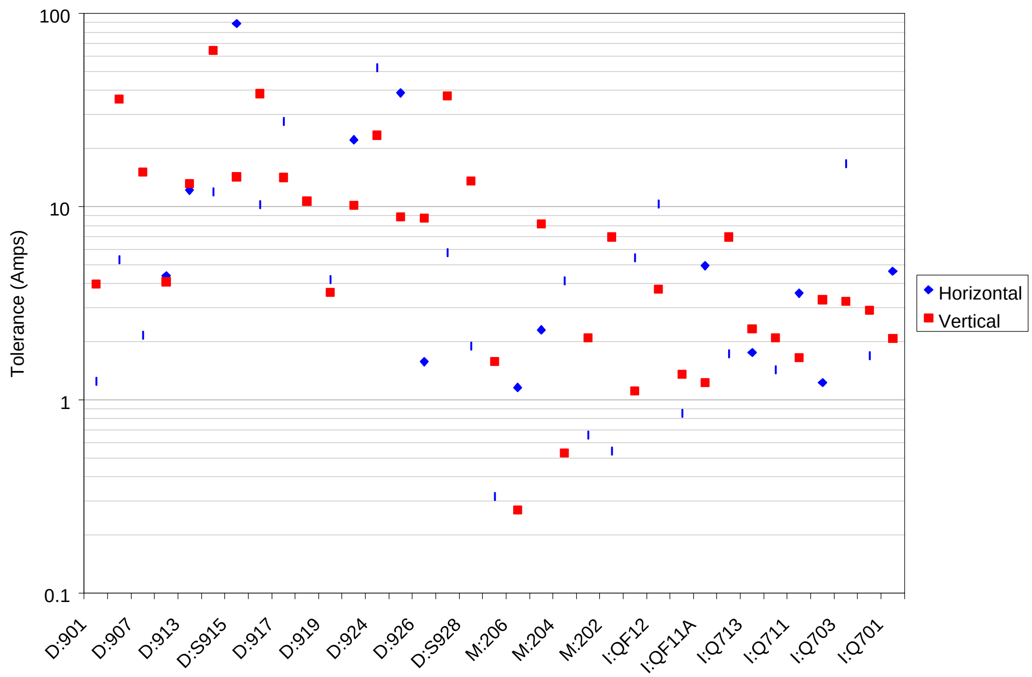

Figure 1. Quadrupole current tolerances for a 10% emittance growth due to focussing errors.

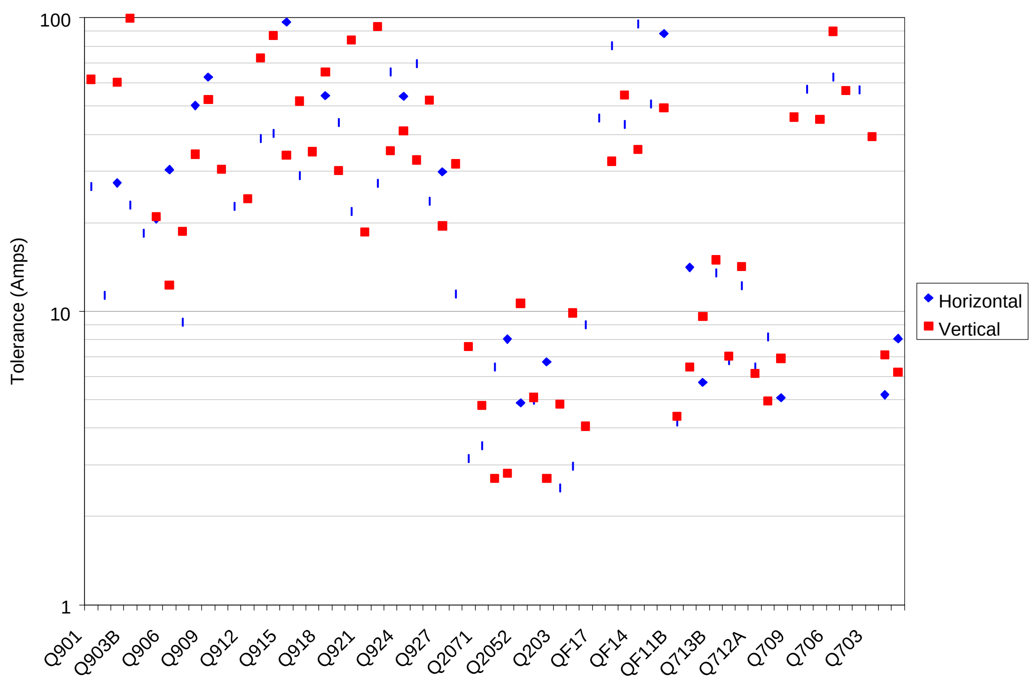

Figure 2. Quadrupole current tolerances that give a 1 p-mm-mrad (normalized, 95%) emittance growth due to quad steering for a 1mm trajectory offset.

Power Supply	Nominal Amps	ΔI Horz Amps	ΔI Vert Amps
D:901	373.87	1.25	3.99
D:903	509.00	5.33	36.04
D:907	87.39	2.17	15.13
D:909	161.48	4.39	4.08
D:913	121.97	12.18	13.19
D:914	185.50	11.94	64.31
D:S915	2.88	88.93	14.26
D:916	264.79	10.25	38.42
D:917	365.31	27.68	14.18
D:S917	50.00	0.00	10.68
D:919	100.53	4.19	3.61
D:S919	8.24	22.23	10.16
D:924	160.08	52.38	23.40
D:S925	16.39	38.77	8.88
D:926	354.04	1.57	8.72
D:S926	39.13	5.79	37.56
D:S928	1.36	1.90	13.61
M:207	22.68	0.32	1.58
M:206	15.05	1.16	0.27
M:205	3.36	2.30	8.17
M:204	19.91	4.15	0.53
M:203	7.54	0.66	2.10
M:202	-5.86	-0.54	-6.97
M:201	10.81	5.44	1.12
I:QF12	99.44	10.30	3.75
I:QF11B	18.14	0.85	1.36
I:QF11A	33.24	4.94	1.23
I:Q714	17.55	1.73	6.97
I:Q713	13.76	1.76	2.33
I:Q712	11.30	1.44	2.10
I:Q711	16.44	3.56	1.66
I:Q710	15.19	1.23	3.30
I:Q703	216.19	16.74	3.25
I:Q702	15.25	1.70	2.90
I:Q701	13.81	4.63	2.08

Table 1. Quadrupole current tolerances for a 10% emittance growth due to focussing errors.

Quad Name	Power Supply	Nominal Amps	Length meters	Gradient meters^{-2}	ΔI Horz Amps	ΔI Vert Amps
Q901	D:Q901	373.87	0.701	-0.507	26.63	61.67
Q902	D:Q901	373.87	0.828	0.508	11.34	115.84
Q903A	D:Q903	509.00	0.828	-0.631	27.34	60.28
Q903B	D:Q903	509.00	0.828	-0.631	23.01	99.64
Q904	D:Q901	373.87	0.640	0.506	18.42	196.59
Q905	D:Q901	373.87	0.701	0.507	20.65	20.99
Q906	D:Q901	373.87	0.828	-0.508	30.38	12.27
Q907	D:Q907	87.39	1.312	0.123	9.16	18.75
Q908	D:Q907	87.39	0.457	-0.124	50.21	34.28
Q909	D:Q909	161.48	0.457	0.226	62.67	52.72
Q910	D:Q909	161.48	0.457	-0.226	140.31	30.42
Q911	D:Q909	161.48	0.457	0.226	22.77	120.68
Q912	D:Q909	161.48	0.457	-0.226	133.33	24.15
Q913	D:Q913	121.97	0.457	0.172	38.72	72.94
Q914	D:Q914	185.50	0.457	0.260	40.37	86.94
Q915	D:QS915	119.09	0.701	-0.167	96.62	34.01
Q916	D:Q916	264.79	0.701	0.372	29.01	52.04
Q917	D:QS917	315.31	0.457	-0.435	289.76	34.98
Q918	D:Q917	365.31	0.457	0.494	54.14	65.45
Q919	D:QS919	92.29	0.640	-0.130	43.95	30.18
Q920	D:Q919	100.53	0.457	0.142	21.89	83.95
Q921	D:Q919	100.53	0.457	-0.142	122.75	18.63
Q922	D:Q919	100.53	0.457	0.142	27.28	93.21
Q923	D:Q919	100.53	0.457	-0.142	65.45	35.31
Q924	D:Q924	160.08	0.457	0.225	54.01	41.17
Q925	D:QS925	143.69	0.457	-0.202	69.82	32.77
Q926	D:QS926	314.91	0.640	0.433	23.69	52.49
Q927	D:Q926	354.04	0.701	-0.482	29.92	19.54
Q928	D:QS928	352.68	0.828	0.483	11.44	31.78

Table 2a. Quadrupole current tolerances that give a 1 π-mm-mrad (normalized, 95%) emittance growth due to quad steering for a 1mm trajectory offset.

Quad Name	Power Supply	Nominal Amps	Length meters	Gradient meters^{-2}	ΔI Horz Amps	ΔI Vert Amps
Q2072	M:Q207	22.68	3.048	0.037	3.15	7.60
Q2071	M:Q207	22.68	3.048	0.037	3.48	4.78
Q2062	M:Q206	15.05	3.048	-0.025	6.45	2.70
Q2061	M:Q206	15.05	3.048	-0.025	8.04	2.81
Q2052	M:Q205	3.36	3.048	0.009	4.89	10.64
Q2051	M:Q205	3.36	3.048	0.009	4.97	5.09
Q204	M:Q204	19.91	3.048	-0.034	6.73	2.69
Q203	M:Q203	7.54	3.048	0.015	2.50	4.82
Q202	M:Q202	-5.86	3.048	0.010	2.96	9.87
Q201	M:Q201	10.81	3.048	-0.018	8.99	4.06
QF17	I:QF12	99.44	2.134	0.019	45.58	104.79
QF16	I:QF12	99.44	2.134	-0.019	80.17	32.44
QF15	I:QF12	99.44	2.134	0.019	43.27	54.67
QF14	I:QF12	99.44	2.134	-0.019	95.37	35.58
QF13	I:QF12	99.44	2.134	0.019	50.87	133.10
QF12	I:QF12	99.44	2.134	-0.019	88.38	49.34
QF11B	I:QF11B	18.14	3.048	0.030	4.19	4.38
QF11A	I:QF11A	33.24	1.524	-0.054	14.11	6.47
Q714	I:Q714	17.55	3.048	0.029	5.74	9.60
Q713B	I:Q713	13.76	1.524	-0.023	13.50	14.97
Q713A	I:Q713	13.76	3.048	-0.024	6.80	7.02
Q712B	I:Q712	11.30	1.524	0.020	12.20	14.20
Q712A	I:Q712	11.30	3.048	0.020	6.47	6.13
Q711	I:Q711	16.44	3.048	-0.028	8.17	4.95
Q710	I:Q710	15.19	3.048	0.026	5.07	6.91
Q709	I:Q703	216.19	2.134	-0.042	148.82	45.88
Q708	I:Q703	216.19	2.134	0.042	57.10	235.81
Q707	I:Q703	216.19	2.134	-0.042	145.09	45.11
Q706	I:Q703	216.19	2.134	0.042	62.63	89.86
Q705	I:Q703	216.19	2.134	-0.042	145.48	56.58
Q704	I:Q703	216.19	2.134	0.042	56.83	182.09
Q703	I:Q703	216.19	2.134	-0.042	148.46	39.44
Q702	I:Q702	15.25	3.048	0.026	5.19	7.10
Q701	I:Q701	13.81	3.048	-0.023	8.07	6.19

Table 2b. Quadrupole current tolerances that give a 1 π-mm-mrad (normalized, 95%) emittance growth due to quad steering for a 1mm trajectory offset.

Name	s meters	β meters	α	μ radians	D meters	D'
ACCUM	0.000	33.250	-0.534	0.000	0.619	-0.074
Q901	8.411	54.742	-18.680	0.221	-0.011	-0.071
Q902	12.375	214.978	58.420	0.253	-0.248	0.034
Q903A	14.789	44.534	1.302	0.282	-0.208	-0.068
Q903B	15.907	62.860	-27.552	0.305	-0.340	-0.211
Q904	17.333	137.421	2.514	0.319	-0.585	-0.031
Q905	21.093	91.009	37.814	0.349	-0.616	0.205
Q906	22.414	29.949	7.051	0.377	-0.429	0.012
Q907	35.935	122.047	5.243	3.316	-0.243	0.055
Q908	47.422	33.264	0.699	3.499	0.396	0.077
Q909	66.134	21.708	2.145	4.247	1.789	-0.110
Q910	75.452	4.331	-0.699	5.634	0.786	-0.027
Q911	95.628	164.508	9.671	6.464	0.237	-0.053
Q912	110.244	4.796	0.788	7.026	-0.543	-0.107
Q913	125.406	56.545	0.152	9.037	-2.121	0.060
Q914	131.636	52.404	6.382	9.149	-1.701	0.267
Q915	137.958	3.894	0.920	9.626	-0.511	0.098
Q916	150.085	42.858	7.072	11.738	0.055	-0.023
Q917	156.355	1.048	-0.577	13.526	-0.335	-0.187
Q918	161.052	31.279	0.747	14.423	-1.155	0.073
Q919	171.550	22.234	-1.597	14.844	-0.401	0.038
Q920	194.647	175.905	6.231	15.216	0.474	0.008
Q921	220.057	5.595	0.120	16.181	0.680	0.051
Q922	245.480	113.280	2.926	17.652	1.954	-0.076
Q923	270.873	19.680	-0.480	18.229	0.027	-0.073
Q924	278.493	29.228	2.091	18.545	-0.517	-0.021
Q925	282.035	17.456	-0.147	18.706	-0.603	-0.075
Q926	316.411	80.664	21.614	19.697	-1.456	0.459
Q927	322.606	42.906	-27.507	22.722	1.716	1.016
Q928	325.034	208.048	21.566	22.744	3.600	-0.411

Table 3a. Horizontal lattice functions at the antiproton downstream end of the quadrupoles in the AP3-P1 line.

Name	s meters	β meters	α	μ radians	D meters	D'
Q2072	343.804	141.827	-3.172	25.793	-3.884	-0.088
Q2071	347.169	115.560	11.383	25.817	-3.507	0.345
Q2062	352.759	32.402	3.002	25.916	-1.848	0.177
Q2061	356.103	20.857	0.549	26.051	-1.449	0.055
Q2052	376.125	22.046	-0.083	27.168	-0.334	0.067
Q2051	379.463	21.272	0.360	27.320	-0.102	0.072
Q204	392.731	28.229	-2.994	27.984	0.973	0.154
Q203	405.911	147.781	-0.504	28.177	2.815	0.029
Q202	422.992	152.719	4.074	28.286	2.790	-0.095
Q201	451.773	16.396	-0.087	28.956	0.089	-0.082
QF17	489.017	98.981	1.819	30.043	-1.788	0.028
QF16	518.760	31.994	-0.750	30.626	-1.557	-0.066
QF15	548.504	109.845	2.441	31.128	-3.904	0.062
QF14	578.247	22.611	-0.342	31.799	-2.731	-0.081
QF13	607.990	79.467	1.548	32.537	-5.449	0.112
QF12	640.025	26.329	-0.869	33.355	-2.675	-0.033
QF11B	663.246	77.552	5.450	33.821	-3.241	0.256
QF11A	669.386	29.274	0.751	33.957	-1.791	0.101
Q714	709.012	40.923	2.657	35.546	1.927	-0.074
Q713B	711.943	28.657	1.081	35.633	1.759	-0.011
Q713A	715.296	27.911	-1.025	35.757	1.917	0.119
Q712B	718.324	33.281	-0.234	35.855	2.228	0.054
Q712A	721.677	29.217	1.538	35.959	2.198	-0.083
Q711	727.667	19.658	-0.665	36.234	1.949	0.079
Q710	745.152	51.782	2.427	36.748	3.376	-0.147
Q709	762.254	9.292	-0.612	37.761	1.456	0.030
Q708	779.735	63.118	2.798	38.482	2.210	-0.127
Q707	797.215	9.776	-0.450	39.355	0.489	-0.033
Q706	814.695	52.465	2.384	40.138	0.307	-0.017
Q705	832.175	9.724	-0.643	41.140	0.492	0.067
Q704	849.658	63.726	2.870	41.836	1.655	-0.059
Q703	867.138	9.337	-0.430	42.732	0.745	0.023
Q702	884.088	47.991	1.534	43.531	1.000	-0.061
Q701	900.819	20.156	-0.962	44.162	-0.007	-0.055
Q522	914.004	52.041	2.450	44.546	-0.686	-0.001
P150	918.382	33.167	1.861	44.652	-0.691	-0.001

Table 3b. Horizontal lattice functions at the antiproton downstream end of the quadrupoles in the AP3-P1 line.

Name	s meters	β meters	α	μ radians	D meters	D'
ACCUM	0.000	5.573	0.493	0.000	0.000	0.000
Q901	8.411	10.206	2.563	1.407	-0.567	0.103
Q902	12.375	2.060	-1.093	2.911	-0.043	0.114
Q903A	14.789	9.162	1.801	3.396	0.197	0.024
Q903B	15.907	3.354	3.273	3.577	0.179	-0.080
Q904	17.333	1.207	-1.931	5.853	0.075	-0.050
Q905	21.093	88.104	-45.393	6.270	0.000	0.000
Q906	22.414	183.721	8.862	6.280	0.000	0.000
Q907	35.935	29.137	-1.359	6.485	-0.005	-0.001
Q908	47.422	71.385	1.587	6.736	-0.018	0.000
Q909	66.134	30.675	-2.462	7.158	-0.020	-0.002
Q910	75.452	92.107	5.081	7.330	-0.039	0.002
Q911	95.628	5.855	-1.388	9.376	0.000	0.002
Q912	110.244	146.249	6.685	9.886	0.026	-0.001
Q913	125.406	15.936	0.715	10.211	0.013	0.000
Q914	131.636	11.299	-1.194	10.707	0.015	0.002
Q915	137.958	31.425	1.357	11.034	0.026	-0.001
Q916	150.085	13.316	-2.978	11.725	0.013	0.002
Q917	156.355	71.887	7.455	11.919	0.024	-0.003
Q918	161.052	21.398	-0.904	12.045	0.011	0.000
Q919	171.550	47.164	2.208	12.373	0.006	-0.001
Q920	194.647	11.964	-1.436	14.108	-0.016	-0.002
Q921	220.057	242.995	7.965	14.591	-0.066	0.002
Q922	245.480	9.704	0.601	15.151	-0.008	0.002
Q923	270.873	67.623	1.465	16.938	0.037	-0.001
Q924	278.493	50.304	-3.973	17.072	0.033	0.003
Q925	282.035	79.269	2.257	17.127	0.041	-0.001
Q926	316.411	16.430	-4.705	18.717	0.002	0.000
Q927	322.606	100.632	23.926	18.853	0.000	0.000
Q928	325.034	26.987	0.808	18.905	-0.002	-0.001

Table 4a. Vertical lattice functions at the antiproton downstream end of the quadrupoles in the AP3-P1 line.

Name	s meters	β meters	α	μ radians	D meters	D'
Q2072	343.804	24.316	-2.740	19.901	-0.020	-0.003
Q2071	347.169	61.500	-10.075	19.994	-0.034	-0.006
Q2062	352.759	185.353	-4.376	20.043	-0.060	-0.002
Q2061	356.103	171.104	9.498	20.061	-0.058	0.003
Q2052	376.125	4.644	-1.255	22.373	0.179	0.020
Q2051	379.463	20.301	-3.602	22.730	0.254	0.026
Q204	392.731	176.230	7.598	22.924	0.510	-0.029
Q203	405.911	39.862	1.485	23.092	0.138	-0.021
Q202	422.992	13.769	-0.281	23.953	-0.198	-0.019
Q201	451.773	80.489	2.194	24.876	-0.698	0.020
QF17	489.017	18.726	-1.244	26.482	-0.871	-0.064
QF16	518.760	195.460	2.975	26.972	-2.651	0.046
QF15	548.504	68.813	-1.264	27.243	-1.356	-0.010
QF14	578.247	162.451	4.488	27.516	-1.593	0.057
QF13	607.990	11.607	0.160	28.310	0.090	0.058
QF12	640.025	84.455	0.898	29.683	1.723	-0.012
QF11B	663.246	71.119	-5.581	30.028	1.903	0.170
QF11A	669.386	138.982	3.440	30.087	2.777	-0.055
Q714	709.012	14.603	-1.397	31.569	0.618	-0.005
Q713B	711.943	23.322	-1.148	31.725	0.588	-0.026
Q713A	715.296	26.184	0.524	31.855	0.441	-0.063
Q712B	718.324	24.566	-0.350	31.977	0.257	-0.054
Q712A	721.677	32.592	-2.354	32.100	0.092	-0.044
Q711	727.667	53.452	1.366	32.232	-0.164	-0.036
Q710	745.152	27.877	-1.574	32.757	-1.011	-0.113
Q709	762.254	97.780	5.615	33.061	-2.696	0.134
Q708	779.735	3.701	-0.475	34.618	-0.430	0.083
Q707	797.215	101.135	3.348	35.589	0.919	0.001
Q706	814.695	25.487	-0.885	35.973	1.153	0.106
Q705	832.175	64.294	4.023	36.378	2.661	-0.147
Q704	849.658	6.207	-1.138	38.258	0.102	-0.128
Q703	867.138	132.273	4.972	38.850	-1.943	0.047
Q702	884.088	25.682	-0.169	39.178	-1.307	-0.055
Q701	900.819	34.269	1.790	39.714	-1.750	0.102
Q522	914.004	10.050	-0.613	40.590	-0.642	0.015
P150	918.382	18.046	-1.213	40.922	-0.576	0.015

Table 4b. Vertical lattice functions at the antiproton downstream end of the quadrupoles in the AP3-P1 line.

REFERENCES

1. PBAR Note 637, "Phase Space Matching Errors", John Marriner, March 3, 1994.
2. PBAR Note 647, "A Look at the AP2 Beam-line", Keith Gollwitzer, January 8, 2001.
3. PBAR Note 648, "Optimization of the 8 GeV AP3-P1 Lattice for Antiproton Transfers", Dave McGinnis, January 22, 2001
4. PBAR Note 649,"Emittance Growth Due to Steering Errors for 8 GeV AP3-P1 Lattice", Dave McGinnis, January 23, 2001