PBAR NOTE 654
BPM Signal Level Calculation

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February 5, 2001

INTRODUCTION

This note is a short summary of the calculation of the induced signal on a transmission line type BPM pickup plate.

BPM IMPULSE RESPONSE

To find the frequency response of the BPM first find the impulse response. The impulse beam current is:

$$i(t) = Q\delta(t) \tag{1}$$

The induced signal on one of the pickup plates is:

$$v(t) = \frac{s}{2}\left(\frac{Z_o i(t)}{2} - \frac{Z_o i\left(t - \frac{2L}{c}\right)}{2}\right) \tag{2}$$

where $\mathbf{Z_o}$ is the characteristic impedance of the transmission line pickup plate, **L** is the length of the pickup plate, and **s** is a measure of how much of the available image current was intercepted by the pickup plate and is between zero and one. The first factor of 2 is because we are examining only one of the pickup plates. The frequency response is found by taking the Fourier transform of the time domain response. The definition of the Fourier transform is:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \tag{3}$$

and:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}d\omega \tag{4}$$

The Fourier transform of the beam current is:

$$I(\omega) = Q \tag{5}$$

The Fourier transform of the pickup voltage is:

$$V(\omega) = \frac{sQZ_o}{4}\left(1 - e^{-j\omega\frac{2L}{c}}\right) \tag{6}$$

If the wavelength of interest is much longer than the pickup:

$$\omega << \frac{c}{2L}$$

then:

$$V(\omega) \approx j\frac{s\omega L Z_o}{2c}Q \tag{7}$$

The pickup impedance is defined as the ratio of Equation 7 to Equation 5.

$$Z(\omega) = j\frac{s\omega L Z_o}{2c} \tag{8}$$

Since:

$$Z_o = \sqrt{\frac{L_o}{C_o}} \tag{9}$$

and:

$$c = \sqrt{\frac{1}{L_o C_o}} \tag{10}$$

where $\mathbf{L_o}$ and $\mathbf{C_o}$ are the inductance and capacitance per length of the pickup plate, the characteristic impedance of the pickup plate is:

$$Z_o = \frac{1}{cC_o} = \frac{L}{cC_{plate}} \tag{11}$$

Equation 8 becomes:

$$Z(\omega) = j\frac{s\omega}{2C_{plate}}\left(\frac{L}{c}\right)^2 \tag{12}$$

THE BEAM CURRENT SPECTRUM

This note will assume that the beam current consists of a train of bunches. If the train is long enough, than the beam current can be represented as a Fourier series:

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos(n\omega_{RF}t) \tag{13}$$

where the AC components (n>0) are given as:

$$I_n = \frac{2\omega_{RF}}{\pi}\int_0^{\pi/\omega_{RF}} \cos(n\omega_{RF}t)dt \tag{14}$$

where $2\pi/\omega_{RF}$ is the spacing between the bunches. If we assume for simplicity that the bunches are rectangular and have a bunch length $\boldsymbol{\tau}$, then:

$$I_n = \frac{\omega_{RF} Q_B}{n\pi} \frac{\sin\left(\frac{n\omega_{RF}\tau}{2}\right)}{\frac{n\omega_{RF}\tau}{2}} \tag{15}$$

where Q_B is the amount of charge in a bunch. If the bunch length is very short compared to the bunch spacing:

$$\tau << \frac{2\pi}{\omega_{RF}}$$

then the beam current frequency component at the bunch spacing frequency (n=1) is:

$$I_1 \approx \frac{\omega_{RF} Q_B}{\pi} \tag{16}$$

The voltage induced on the pickup plate at this frequency is found using Equation 8.

$$V_1 \approx j\frac{s\omega_{RF} L}{2c} Z_o \frac{\omega_{RF} Q_B}{\pi} \tag{17}$$

or using Equation 12:

$$V_1 \approx j\frac{s\omega}{2C_{plate}} \left(\frac{L}{c}\right)^2 \frac{\omega_{RF} Q_B}{\pi} \tag{18}$$

<u>EXAMPLE</u>

Booster Turn ~ $0.45\text{x}10^{12}$ protons

The Booster has 84 bunches

$Q_B = 5.36\text{x}10^9$ protons

L = 0.15 meters = 6"

C_{plate} = 10 pF

s = 1

$\omega_{RF} = 2\ \pi\ \text{x}\ 53\text{x}10^6$ radians / sec

$Z_o = 50\Omega$

V_1 = 0.378 Volts per plate = 1.5 dBm into 50Ω at 53 MHz