

# Discrepancy between Documentation and Code in TRANSPORT

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## Abstract

A discrepancy between the calculations of the fringe-field correction to the pole-face rotation matrix in the TRANSPORT manual and code are documented.

## 1 TRANSPORT Manual

The TRANSPORT manual lists the following matrix for for a pole-face rotation with fringe field correction<sup>1</sup> :

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{\tan(\beta)}{\rho_0} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\tan(\beta-\psi)}{\rho_0} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\psi = \kappa_1 \left( \frac{g}{\rho_0} \right) \left( \frac{1 + \sin 2\beta}{\cos \beta} \right) \left[ 1 - \kappa_1 \kappa_2 \left( \frac{g}{\rho_0} \right) \tan \beta \right]$$

Definitions:

$\beta$  = Angle of rotation of pole-face.

$\rho_0$  = Bending radius of central trajectory.

$g$  = Total gap of magnet.

$\psi$  = Correction term resulting from spatial extent of fringing fields.

The above formula contains a known typographical error<sup>2</sup> found subsequent to SLAC-91. The term  $\sin 2\beta$  should be<sup>3</sup>  $\sin^2 \beta$ . Keeping only the first-order

<sup>1</sup>D.C. Carey, et al., "Third Order TRANSPORT with MAD Input", (Fermi-Pub-98-310, October 1998), 131,132.

<sup>2</sup>D.C. Carey, private communication

<sup>3</sup>see, for example, David C. Carey, "The Optics of Charged Particle Beams" (Harwood Academic Publishers, 1982), 57, eq. 3.59

term in  $\left(\frac{g}{\rho}\right)$ , one find

$$\psi = \kappa_1 \left(\frac{g}{\rho_0}\right) \left(\frac{1 + \sin^2\beta}{\cos\beta}\right)$$

In the TRANSPORT manual the vertical correction, assuming on-momentum particles, is

$$R_{43} = \left(\frac{-1}{\rho}\right) \tan(\beta - \psi) \quad (1)$$

## 2 TRANSPORT Program

Whereas the manual defines  $R_{43}$  as above, TRANSPORT uses the following algorithm to evaluate the variable<sup>4</sup>.

```
      SUBROUTINE FRINGE
C
C   CALCULATES FIRST-ORDER TRANSFER MATRIX FOR BENDING MAGNET
C   FRINGING FIELD
C
C   LIST OF COMMON BLOCKS
C
      INCLUDE 'ELM2A.CIN'
      INCLUDE 'ELM2B.CIN'
      INCLUDE 'ELM4A.CIN'
      INCLUDE 'ELM16A.CIN'
      INCLUDE 'R.CIN'
C
C   LOCAL VARIABLES
C
      REAL  CB, SB2, TB1, TB2, TCOR
C-----
      TB = TAN(BE)
      TB2 = TB*TB
      CB = COS(BE)
      SB = 1.0/CB
      SB2 = SB*SB
      TCOR = 2.0*H0*APB(2)*LAYL
      TB1 = TB - TCOR*SB*(SB2 + TB2)
      R(2,1) = H0*TB
      R(4,3) = - H0*TB1
      LAYK = LAYKI
C
```

<sup>4</sup>D.C Carey, file TRM.F, 19990614.

500 RETURN  
END

Making the proper identification of variables, the "FRINGE" formula for element 4,3 of the pole face rotation becomes:

$$\begin{aligned}
R_{43} &= \left(\frac{-1}{\rho}\right) \left[ \tan\beta - \left(\frac{\kappa_1 g}{\rho \cos\beta}\right) \left(\frac{1}{\cos^2\beta} + \tan^2\beta\right) \right] \\
&= \left(\frac{-1}{\rho}\right) \left[ \tan\beta - \left(\frac{\kappa_1 g}{\rho \cos\beta}\right) \left(\frac{1}{\cos^2\beta} + \frac{\sin^2\beta}{\cos^2\beta}\right) \right] \\
&= \left(\frac{-1}{\rho}\right) \left[ \tan\beta - \left(\frac{\kappa_1 g}{\rho \cos\beta}\right) \left(\frac{1 + \sin^2\beta}{\cos^2\beta}\right) \right] \\
&= \left(\frac{-1}{\rho}\right) \left[ \tan\beta - \left(\frac{\kappa_1 g}{\rho \cos^2\beta}\right) \left(\frac{1 + \sin^2\beta}{\cos\beta}\right) \right]
\end{aligned}$$

Recalling the definition of  $\psi$ , this becomes:

$$R_{43} = \left(\frac{-1}{\rho}\right) \left[ \tan\beta - \frac{\psi}{\cos^2\beta} \right] \quad (2)$$

Clearly, equations 1 and 2 yield different values.

It is also worth noting that, although the TRANSPORT manual makes reference to higher order correction, no provisions are included in the source code.

### 3 Summary

Regarding TRANSPORT, the program does not calculate the fringe-filed correction to a pole-face rotation as one would expect based upon the documentation.