Comparison of the Coupled-Bunch Mode Theory to Experimental Observations in the Fermilab Booster

K. C. Harkay
Purdue University, West Lafayette, IN 47907 *

P. L. Colestock
Fermi National Accelerator Laboratory, P.O. Box 500, MS 341, Batavia, IL 60510 **

Abstract
The well-known longitudinal coupled-bunch mode theory is reviewed and evaluated including finite bunch length effects and Landau damping for the parameters of the Fermilab Booster. Predictions of mode growth rates are found to be in general good agreement with experimental observations, both temporally and in frequency space. The inclusion of Landau damping in the stability analysis is required to achieve overall agreement with the observed unstable mode spectrum. Particle simulation using the ESME code are carried out to describe observations of large amplitude oscillations and saturation effects near the end of the acceleration cycle. Finally, a model of the emittance growth, which is valid for growth rates slow with respect to mode frequencies, is explored.

I. INTRODUCTION

Even after installation of RF cavity mode dampers suppressed the long-observed longitudinal coupled-bunch instability [1], questions remained as to whether the behavior scaled as predicted by the theory. Also, the details of the longitudinal emittance growth scaling were unclear. First, we describe the comprehensive comparison of Booster data with the predictions of the linear coupled-bunch mode theory. While there is strong evidence that nonlinear effects are important, we wished to study the unstable mode growth data quantitatively at least to determine the regime for which the linear theory is valid. A rigorous test of and modification of some of the assumptions used in the literature is required for proper application to the Booster. Solutions are found numerically using standard algorithms. It is found that including self-consistently the effects of the beam momentum spread in the nonlinear RF potential (Landau damping) is essential to accurately describe the unstable beam behavior.

The linear theory is completely inadequate in explaining the emittance growth resulting from the instability. Instead, a fully nonlinear simulation is invoked using the longitudinal particle tracking code ESME developed at Fermilab to study the response of the beam in the presence of a high-Q driving impedance. Subsequent analyses of the results produce an qualitative scaling of the emittance growth and a deepened understanding of the subtleties and sensitivites of the instability on various parameters.

* Current address: Fermi National Accelerator Laboratory, same as above.
** Work supported by the U.S. Department of Energy under Contract No. DE-AC02-76CHO3000.

I. MEASUREMENTS

Three quantities were measured in the Booster for use in the comparison with the theory. First, the impedance due to the RF cavity higher-order modes (HOM), which drive the instability according to the theory, was measured. [2] Because the RF cycles in 33 msec from 30 to 53 MHz, many of the HOMs also tune, so data were recorded corresponding to several times through the cycle. The beam fluctuation spectra were obtained by detecting the signal from a wideband resistive wall monitor and performing an FFT using a TEK DSA 602. An example of a typical spectrum may be seen in [1]. Before suppressing the instability through the recent installation of RF cavity mode dampers, strong oscillations were seen in coupled-bunch mode (wave) numbers around n=16 and 48 (of a possible 84). The spectra were recorded at several times through the cycle and the unstable mode amplitudes were extracted. We see the measured growth of mode n=16 plotted in Fig. 2. Finally, the full (95%) bunch lengths τ_L were measured through the cycle in order to calculate the synchrotron frequency spread.

II. LINEAR THEORY

We begin with the linearized Vlasov equation in polar coordinates (r, θ) [3]

$$-i\Omega f_1 + \omega_s \frac{\partial f_1}{\partial \theta} + \frac{\eta \omega_0}{p} F\left(\phi_0 + r\cos\theta\right) \frac{\sin\theta}{\omega_s} f_0 = 0 \qquad (1)$$

where we assume the particle distribution function f, normalized to unity, may be separated into a stationary and perturbed part given by

$$f(r,\theta,t) = f_0(r) + f_1(r) e^{i\theta} e^{-i\Omega t}, \text{ with } |f_0| >> |f_1| \qquad (2)$$

In this analysis, we consider a pure dipole oscillation only, ie. m=1, and no mode coupling. In (2), Ω are the normal modes of the instability. This quantity is complex, therefore a positive $\mathrm{Im}(\Omega)=\Omega_i$ will lead to growth of the perturbation and $\mathrm{Re}(\Omega)=\Omega_r$ gives a frequency shift. The force F in (1) is the self-induced force on the beam due to its wake fields in the beamline environment. This force may be written as the sum of products of the Fourier components of the charge density and the impedance. After substituting F and summing over all bunches, we arrive at

$$(\Omega-\omega_s(r))f_1(r) = -i\frac{\eta\omega_0{}^3 h}{\beta^2 E}\frac{1}{r}\frac{\partial f_0(r)}{\partial r}\sum_k \frac{Z_k(\omega)}{k} J_1(kr) \times \int_0^\infty r'dr' f_1(r') J_1(kr') \quad (3)$$

where J_1 is the Bessel function and Z_k the impedance due to the RF cavity higher-order modes at the frequencies $\omega = k\omega_0 + \omega_s + \Omega$. The index $k = hp + n$, where h is the harmonic number, p the RF harmonic, and n the coupled-bunch mode number. Multiplying by $r J_1(k'r)/(\Omega - \omega_s(r))$ and integrating both sides over r leads to the dispersion relation

$$1 = -i\frac{\eta\omega_0{}^3 h}{\beta^2 E}\sum_{k,k'}\frac{Z_k(\omega)}{k}\int_0^\infty dr \frac{\frac{\partial f_0(r)}{\partial r} J_1(kr) J_1(k'r)}{(\Omega-\omega_s(r))} \quad (4)$$

Normally, we are interested in solving (4) for each coupled-bunch mode n=0,1,...,h, to find those eigenfrequencies Ω which are unstable. As this is an infinite dimension matrix equation, a number of simplifications are generally made. In the literature, the small-argument expansion is substituted for one or both $J_1(kr)$ terms. This is the short bunch approximation: kr is the ratio of mode amplitude to perturbing wake field wavelength. For the Booster, however, kr is not small. Also, to allow analytical solutions, ω_s is often assumed constant, so that Landau damping can be neglected. We studied both regimes, constant ω_s and $\omega_s(r)$, to examine the influence of Landau damping.

For a constant $\omega_s = \omega_{s_0}$, the denominator in (4) may be pulled out of the integral. In the Booster, there are two unstable modes, each driven by two RF cavity parasitic modes. The mode around n=16 is driven by 169 and 220 MHz (RF order p=3,4). Mode n=48 is driven by 83 and 345 MHz (p=1,6). For each n, (4) becomes a 2x2 matrix equation with solutions given by $|1 - M_{ij}| = 0$. For a gaussian particle distribution, the matrix elements may be written

$$M_{ij} = i\frac{1}{\Delta\Omega}\frac{\eta I_0 e}{2\pi\beta^2 E}\frac{\omega_0{}^2}{\omega_{s_0}{}^2}\frac{1}{L}\frac{Z_{k_i}}{k_i} e^{-\frac{1}{2}(k_i{}^2+k_j{}^2)L^2} I_1\left(k_i k_j L^2\right) \quad (5)$$

where $\Delta\Omega = \left(\Omega - \omega_{s_0}\right)$, $L = \frac{1}{2}\omega_o\tau_L$ is the bunch half length in radians and I_1 are modified Bessel functions. The matrix elements are evaluated for n=16,48 at different times t through the cycle using the measured impedance Z_k. In each case, the larger $\Omega(t)$ of the two solutions is taken to dominate.

In the case of Landau damping, the frequency spread $\omega_s(r)$ must be included. We use the approximation [3]

$$\omega_s(r) = \omega_{s_0} + \Delta\omega_s \approx \omega_{s_0}\left(1 - \frac{r^2}{16}\right), \quad r = hL \quad (6)$$

We eliminate the sum by choosing the dominant Z_k only and write

$$1 = i\frac{\eta I_0 e}{2\pi\beta^2 E}\frac{\omega_0{}^2}{\omega_s{}^2}\frac{16}{\omega_s h^2 L^4}\frac{Z_k}{k}\int_0^\infty dx \frac{e^{-x} J_1{}^2\left(kL\sqrt{2x}\right)}{x-y}$$

$$y = -\frac{8}{\omega_s h^2 L^4}\Delta\Omega \quad (7)$$

$$\Delta\Omega = \left(\Omega_r - \omega_{s_0}\right) + i\Omega_i$$

The integral in equation (7) is solved numerically, fixing Ω_i and varying Ω_r. The curves can be plotted in the complex Z-plane as shown in Fig 1. This example corresponds to Booster parameters at t=29 msec in the cycle, ω_s=2π(2.2) kHz, $\Delta\omega_s$=2π(78) Hz. The solution for $\Omega(t)$ is found graphically by finding the intersection of one of the curves with the measured Z_k/k. If the impedance falls inside the innermost curve, the system is stable. For a Z_k/k = (1.4, 2.0) kΩ as plotted with a large dot, the growth rate predicted is 0.3 msec^{-1} without Landau damping and only 0.1 msec^{-1} with.

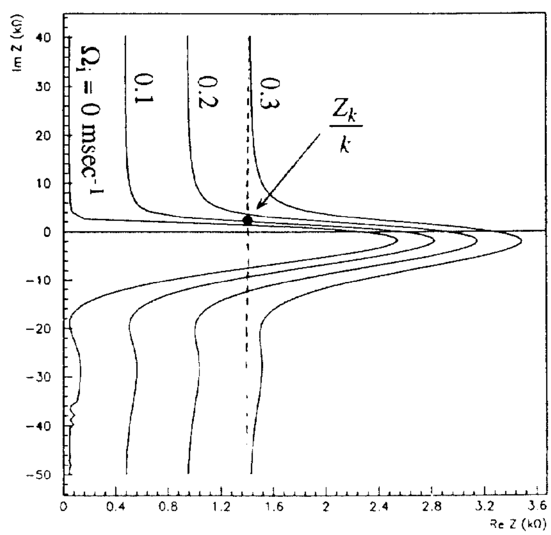

Fig.1. Instability curves showing graphical solution of growth rate.

We may use a WKB approximation, since $\Omega(t)$ is a slow function of time, to get the integrated growth of the instability. The coupled-bunch mode amplitude $\psi(t)$ is calculated using

$$\frac{\psi(t)}{\psi_0(t_0)} = \exp\left(-\int_{t_0}^{t}\left|\Omega_i(t)\right|dt\right) \quad (8)$$

The result, comparing the measured dipole coupled-bunch mode amplitude for n=16 in the Booster with the growth predicted by linear instability theory both without and with Landau damping, is shown in Fig 2.

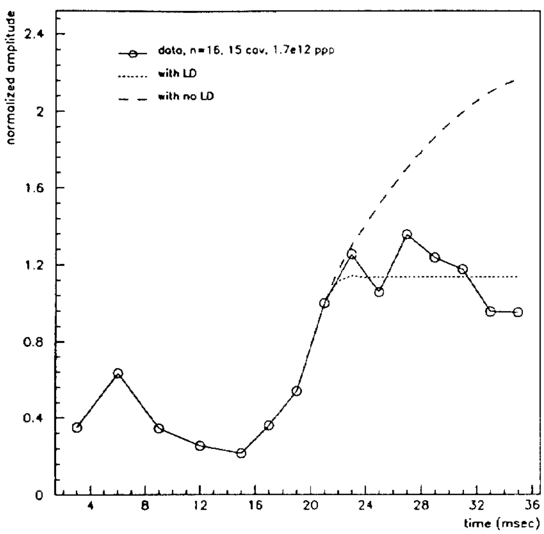

Fig. 2. Linear coupled-bunch theory vs. Booster data, *n*=16.

III. SIMULATION

We used the longitudinal particle tracking code ESME [4] to simulate the Booster using the measured RF cavity HOMs [2]. A full ring of 84 bunches (46k macro particles) were tracked driven in separate runs by 83 and 220 MHz impedances modelled as LRC resonators. The results depend strongly on the initial particle distribution. We reproduced the observed dipole amplitude and emittance growth with a gaussian distribution tracked through transition. In each case, the bunches begin to oscillate rather coherently until, at different radii, they filament in the nonlinear RF potential. The results are shown in Fig. 3. The bunches clearly begin to filament at a larger amplitude on the left. The unstable behavior depends on both the growth rate Ω_i and the frequency shift Ω_r which, for 83 MHz, are a factor of 2 and 100 larger, respectively. The unstable impedance parameter space was explored to determine a scaling for predicting the final maximum amplitude. Additional details are discussed in [5] and [6].

IV. DISCUSSION

Results from the linear coupled-bunch instabilty theory show good quantitative agreement with the measured growth despite the large amplitude ocsillations. It is shown that Landau damping must be invoked to predict the observed saturation. Two additional corrections should be noted. First, the expression for the radial dependence of the frequency ω_s in (6) is strictly true only for a stationary bucket, ie. when $\phi_s = 0$. A more general expression gives a moving bucket $\Delta\omega_s$ scaling of $\left(1+\frac{5}{3}\tan^2\phi_s\right)$[7]. This result, using the same analysis as that leading to (7), predicts growth rates reduced by up to a factor of ten near transition energy, where $\phi_s = 65^\circ$. This seems, however, inconsistent with observations. Second, the measured stationary particle distributions extracted from the envelope of the RF harmonics in the beam spectra may be used instead in the calculations.

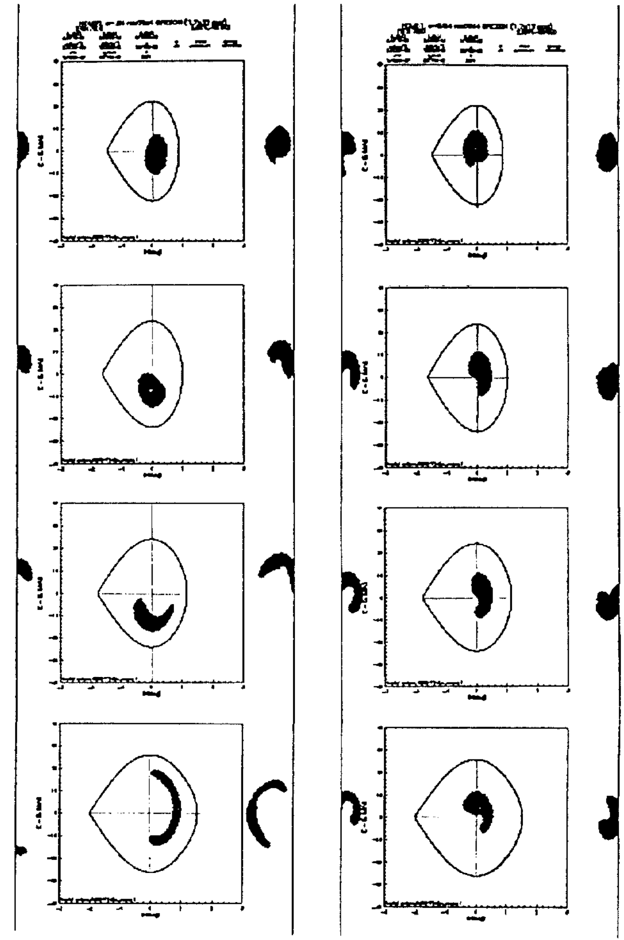

Fig. 3. Phase space plots from ESME simulation for an impedance at 83 MHz (left) and 220 MHz (right) for the last 4 msec in the Booster cycle. The axis scales are $\Delta E = \pm 40\,MeV$ and $\Delta\phi = \pm 3^\circ$.

The authors would like to thank A. Gerasimov and D. Wildman for productive discussions on emittance growth scaling and RF cavity measurements, respectively.

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