Head-Tail Modes for FNAL Booster

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The FNAL Booster accelerator is a proton synchrotron, originally designed and constructed in the begining of 1970's to match the beam from the linear accelerator to the Fermilab Main Ring. The major motivation for the construction was the cost reduction of the whole accelerator facility, as well as the beam quality improvement.

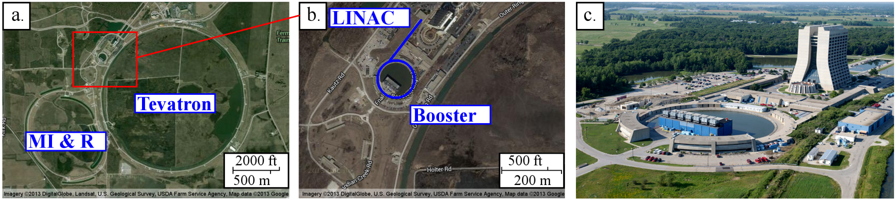


Figure: (a.), (b.) Satellite images of the Fermilab site showing Linear Accelerator (LINAC), Booster, Main Injector (MI), Recycler (R) and Tevatron ring. (c.) Photo of the Fermilab Willson Hall, LINAC and Booster.

Since the 70's the whole accelerator complex has undergone many changes. At the present time, Booster accumulates the 400 MeV proton beam from the LINAC and then gives an intermediate boost to the beam energy. Booster was build as a fast cycling machine operating at 15 Hz which goes through repeated acceleration cycles delivering extracted 8 GeV beam pulses (referred to as a batch) to different experiments or filling the Main Injector ring which is about seven times larger.

A multiple turn injection system increases the Booster intensity; it allows to stack successive turns of LINAC beam layered on top of each other. LINAC provides Booster with 400 MeV debunched H^- ion beam. The H^- ions and circulating beam passes through the stripping foil, which removes electrons of the ions and made of a thin layer of carbons. Operationally, the practical limit for maximum intensity is about 7 to 8 turns; fractional turns are not used normally.

1.2 Booster Lattice Parameters

Booster lattice consists of 96 combined function magnets which are arranged in 24 superperiods of the FOFDOOD-type cells; all magnets are combined function magnets which bend the beam and focus it either horizontally or vertically.

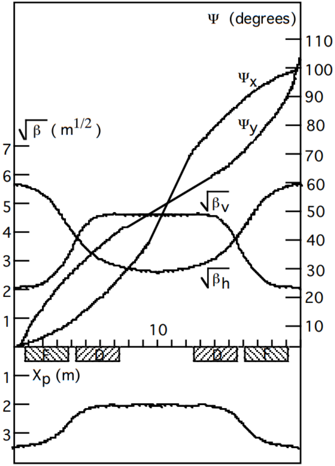


Figure: Top plot shows beta functions and phase advances in one Booster superperiod (x and y are horizontal and vertical degrees of freedom respectively). Bottom plot shows horizontal dispersion.

Table: Booster ring and beam parameters.

Transverse motion parameters		
Max horizontal beta function, $\beta_{x,\ \mathrm{max}}$	(m)	33.7
Max vertical beta function, $\beta_{y,\ \mathrm{max}}$	(m)	20.5
Betatron tune, $\nu_{x,y}$		6.7
Phase advance per cell, $\mu_{x,y}$	(°)	96
Maximum dispersion	(m)	3.2
Longitudinal motion parameters		
Harmonic number, h		84
Injection RF frequency, f_{RF}	(MHz)	37.77
Extraction RF frequency, f_{RF}	(MHz)	52.81
Maximum RF voltage	(MV)	0.86

Booster structure		
Cell type		FOFDOOD
# of bend magnets		96 (4 per cell)
# of superperiods		24
Magnetic field at injection, B_0	(KGauss)	0.74
Magnetic field at extraction, B_0	(KGauss)	7
Bend magnet length	(m)	2.9
Circumference, Π	(m)	$2\pi \times 74.47$
Cycle time, T	(sec)	$1/15$

Beam parameters		
Transverse normalized emittance, $\epsilon_{x,y}$	(mm·rad)	12π
Longitudinal emittance, $\varepsilon_{\rm long}$	(eV·sec)	0.25
Typical bunch intensity,		3×10^{10}
Injection kinetic energy, $E_{\rm kin}$	(MeV)	400
Transition kinetic energy, $E_{\rm kin}$	(GeV)	4.17
Extraction kinetic energy, $E_{\rm kin}$	(GeV)	8

2.1 Head-Tail Modes for Strong Space Charge

The general framework for the description of head-tail modes for strong space charge beam was developed in [Burov, 2009]. The theory is valid when the space charge tune shift in the 3D center of the bunch is much larger than both the synchrotron tune, Q_s, and the wake-driven coherent tune shift, Q_w:

$$Q_{\max} \gg Q_s, Q_w.$$

In this case the force on a particle due to a space charge is just proportional to its offset from the local beam centroid and a single-particle equation of motion in a no-wake case can be written in rigid-slice approximation.

Single-particle equation of motion

$$\dot{x}_i(\theta) = iQ\left[\tau_i(\theta)\right]\left\{x_i(\theta) - \bar{x}\left[\theta, \tau_i(\theta)\right]\right\} - i\zeta v_i(\theta)x_i(\theta)$$

where

- θ is the time in radians (operator $(\dot{\ldots}) \overset{\mathrm{def}}{=} \mathrm{d}/\mathrm{d}\,\theta$),
- $x_i(\theta)$ is a slow variable related to the betatron offset of the i-th particle as $X_i(\theta) = x_i(\theta)e^{-iQ_b\theta}$,
- Q_b is a betatron frequency,
- $\bar{X}(\theta, \tau)$ is an offset of the beam slice where τ is a distance along the bunch in radians,
- $v_i \overset{\mathrm{def}}{=} \dot{\tau}_i(\theta)$ is the velocity of i-th particle,
- $Q(\tau)$ is the space charge tune shift as a function of the offset τ and the action variables of the particle i,
- ζ is the effective chromaticity defined as $-\xi/\eta$ with $\xi = \frac{\mathrm{d}\,Q_b}{\mathrm{d}\Delta p/p}$ being the conventional chromaticity, and $\eta = \gamma_t^{-2} - \gamma^{-2}$ the slippage factor.

The substitution of

$$x_i(\theta) = y_i(\theta) e^{-i\zeta\tau_i(\theta)},$$

allows to exclude the chromaticity term:

$$\dot{y}_i(\theta) = iQ\left[\tau_i(\theta)\right]\left\{y_i(\theta) - \bar{y}\left[\theta, \tau_i(\theta)\right]\right\}.$$

Further simplification allows to write the second order differential equation for strong space charge eigenfunctions:

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}\tau}\left(u^2(\tau)\frac{\mathrm{d}\bar{y}(\tau)}{\mathrm{d}\tau}\right) + \nu Q\bar{y}(\tau) = 0},$$

with

$$u^2(\tau) = \frac{\int_{-\infty}^{\infty} v^2 f(v,\tau)\,\mathrm{d}v}{\int_{-\infty}^{\infty} f(v,\tau)\,\mathrm{d}v}.$$

2.2 Head-Tail Modes for Gaussian Bunch

Consider the case of Gaussian bunch with longitudinal distribution function defined as

$$f(v,\tau) = \frac{N_\mathrm{b}}{2\pi\sigma u} e^{-v^2/2u^2 - \tau^2/2\sigma^2}.$$

Its substitution into equation of motion leads to the SL problem:

$$\begin{cases} \boxed{\bar{y}''(\tau) + \nu\, e^{-\tau^2/2}\, \bar{y}(\tau) = 0}, \\ \bar{y}'(\pm\infty) = 0. \end{cases}$$

The natural system of units is employed: the distance τ is measured in units of the rms bunch length σ and eigenvalues ν_k is measured in units of

$$\frac{u^2}{\sigma^2 Q_\mathrm{eff}(0)} = \frac{Q_s^2}{Q_\mathrm{eff}(0)}.$$

Boundary conditions are selecting a sequence of discrete real eigenvalues which can be ordered such that

$$\nu_0 < \nu_1 < \ldots < \nu_k < \ldots \to \infty.$$

Corresponding to each eigenvalue, ν_k, there is a unique eigenfunction $\bar{y}_k(\tau)$, which has exactly k zeros. The full set of eigenfunctions form a full orthogonal basis which can be also normalized so

$$\int_{-\infty}^{\infty} \rho(\tau)\bar{y}_i(\tau)\bar{y}_j(\tau)\,\mathrm{d}\tau = \delta_{ij},$$

where δ_{ij} is a Kronecker delta and

$$\rho(\tau) = \int_{-\infty}^{\infty} f(v,\tau)\,\mathrm{d}v = \frac{1}{\sqrt{2\pi}}\,e^{-\tau^2/2}$$

is the normalized line density of the beam.

In order to get the numerical solution, one can consider two identical SL problems with cos- and sin-like boundary conditions

$$\begin{cases} \bar{y}_{\rm c}'' + \nu\, e^{-\tau^2/2}\, \bar{y}_{\rm c} = 0, \\ \bar{y}_{\rm c}(0) = 1,\ \ \bar{y}_{\rm c}'(0) = 0, \end{cases} \qquad \begin{cases} \bar{y}_{\rm s}'' + \nu\, e^{-\tau^2/2}\, \bar{y}_{\rm s} = 0, \\ \bar{y}_{\rm s}(0) = 0,\ \ \bar{y}_{\rm s}'(0) = 1. \end{cases}$$

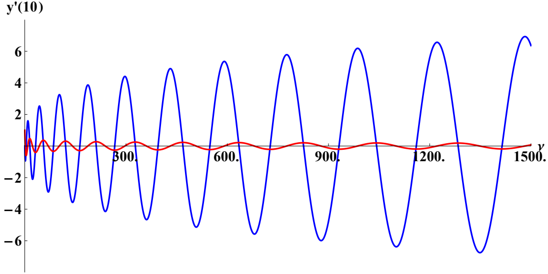


Figure: Numerically obtained value of $\bar{y}_{{\rm c},s}'(\tau_{\max})$ as a function of frequency for $\tau_{\max} = 10$. Blue and red curves correspond to even and odd SL problems respectively. Step in frequency used in calculation is $\Delta\nu = 10^{-2}$.

Eigenfunctions tend to a constant at the tail of bunch, $\bar{y}_k(\infty) \neq 0$, while their derivatives tend to zero with asymptotic behavior such that

$$\bar{y}'(\tau) \approxeq \nu\,\bar{y}(\infty)\frac{e^{-\tau^2/2}}{\tau}.$$

k	0	1	2	3	4	5	6
ν_k	0	1.34	4.32	8.9	15.05	22.79	32.1
ω_k	1	1.47	1.19	3.6	1.19	5.71	1.19
$\lvert\bar{y}_k(\infty)\rvert$	1	1.63	1.99	2.27	2.51	2.72	2.92
k	7	8	9	10	11	12	13
ν_k	42.98	55.44	69.47	85.08	102.26	121.01	141.34
ω_k	7.83	1.19	9.94	1.19	12.05	1.19	14.16
$\lvert\bar{y}_k(\infty)\rvert$	3.1	3.26	3.42	3.57	3.71	3.84	3.97

Table: Eigenvalues, eigenweights and asymptotic values for the first 14 SSC modes of the Gaussian bunch.

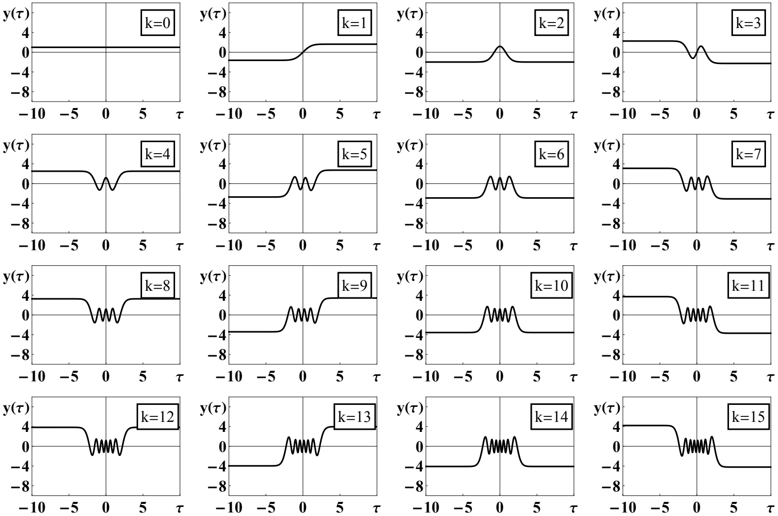


Figure: The first 16 eigenfunctions of the Gaussian bunch, $\bar{y}_k(\tau)$. The modes do not depend on the chromaticity, except the common head-tail phas factor $e^{-i\zeta\tau}$.

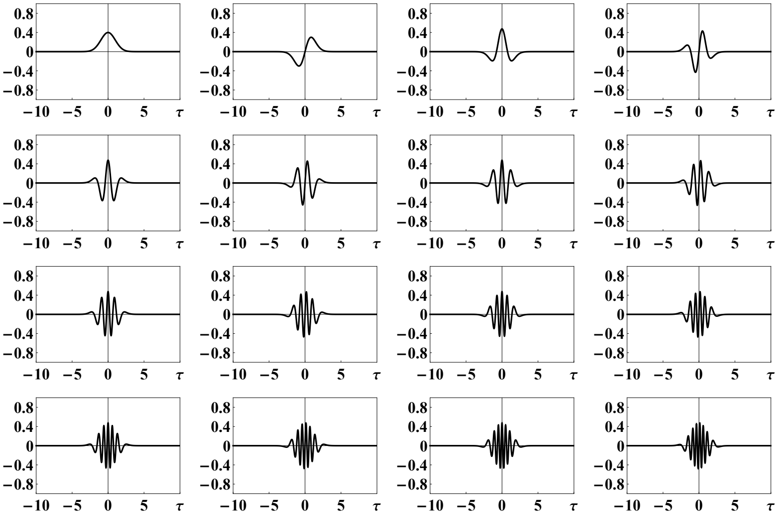
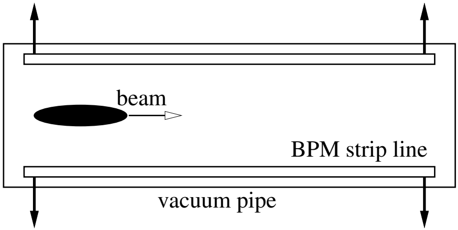


Figure: The first 16 zero chromaticity local dipole moments of the Gaussian bunch, $\rho(\tau)\bar{y}_k(\tau)$.

3.1 Beam Position Monitoring

Apparatus

The most common method of a non-intercepting particle beam monitoring is to couple to the electromagnetic field of the beam. The conventional BPM is a pair (or two pairs) of electrodes on which the signals are induced: **detailed knowledge of how the magnetic and electric fields depend on the beam position allows accurate determination of the beam position and sometimes the internal beam structure**.



BPM Signal Formation

Consider a current of a Gaussian-shaped beam bunch containing N particles of a charge e:

$$I_b(t) = \frac{eN}{\sqrt{2\pi}\sigma} \exp\left[-\frac{t^2}{2\sigma^2}\right].$$

For a centroid beam the induced wall current density at the beam pipe is uniformly distributed (in azimuthal direction) and simply

$$i_w(t) = -\frac{1}{2\pi R} I_b(t).$$

When the beam is displaced from the center to the position (r, θ) the wall current becomes

$$i_w(t, R, \phi_w) = -\frac{1}{2\pi R} I_b(t) \left(1 + 2\sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^n \cos n(\phi_w - \theta)\right).$$

The last expression let one to express a beam displacement $x = r\cos\theta$ as a function of currents on opposite strip lines as:

$$x \propto \frac{A-B}{A+B} + \text{Higher-order terms}$$

When the beam pulse approaches the upstream end of the electrode it launches TEM waves in two directions: one signal goes out the upstream port to the electronics, while the other wave travels down the outside surface of the electrode to the downstream port and induce a similar signal with opposite polarity to the first one. The net result for the Gaussian beam has a form of

$$V(t) \propto \exp\left[-\frac{(t+\tau)^2}{2\sigma^2}\right] - \exp\left[-\frac{(t-\tau)^2}{2\sigma^2}\right]$$

3.2 Experiment Setup

The experimental data gathered for the regular operational setup of the Booster. Three different intensities provided by 3, 7 and 14 turns of injection from LINAC have been studied; the corresponding values of number of particles injected into the machine are 0.63, 2.01 and 4.26×10^{12}.

One can note the distinguishing lost of particles due to collective instability at the transition for highest intensity.

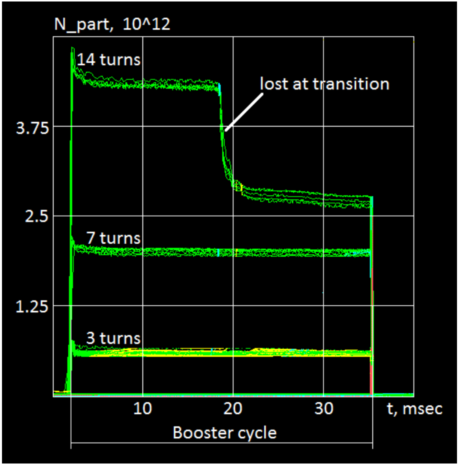


Figure: Beam intensities for 3, 7 and 14 turn settings for Booster cycle.

Data has been obtained for two settings of chromaticity: the post-transition "normal" horizontal chromaticity that is about zero or a bit positive and post-transition with negative horizontal chromaticity.

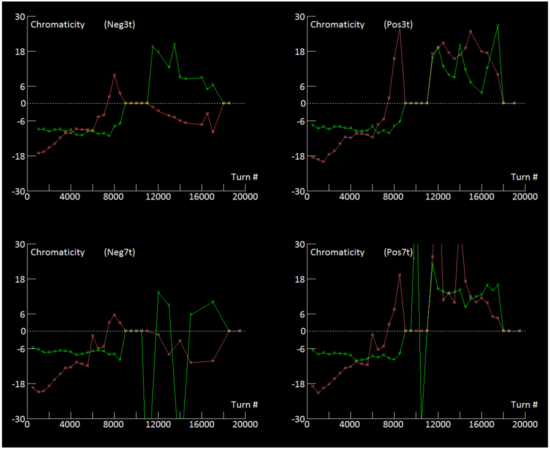


Figure: Measured chromaticity for 3 and 7 turns. Red and green curves correspond to horizontal and vertical degrees of freedom.

In order to excite head-tail modes all bunches were pinged every 100 turns in both directions.

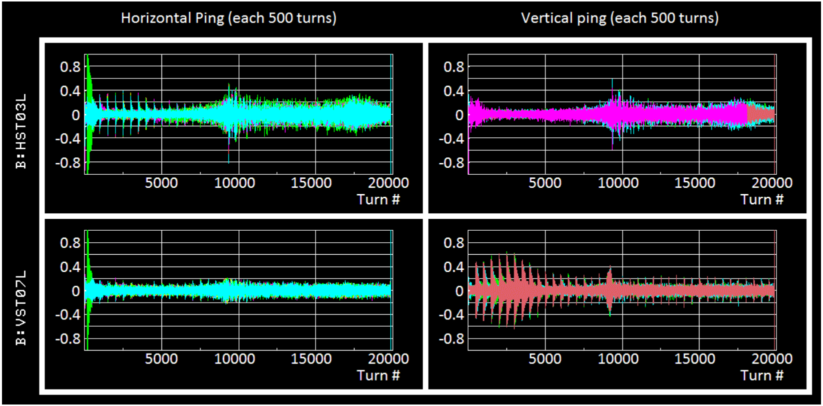


Figure: Top and bottom rows of figures show the signal obtained from horizontal plates of a BPM located at long section 03 and vertical plates of another BPM located at long section 07. Left and right columns are correspond to the horizontal and vertical pining respectively.

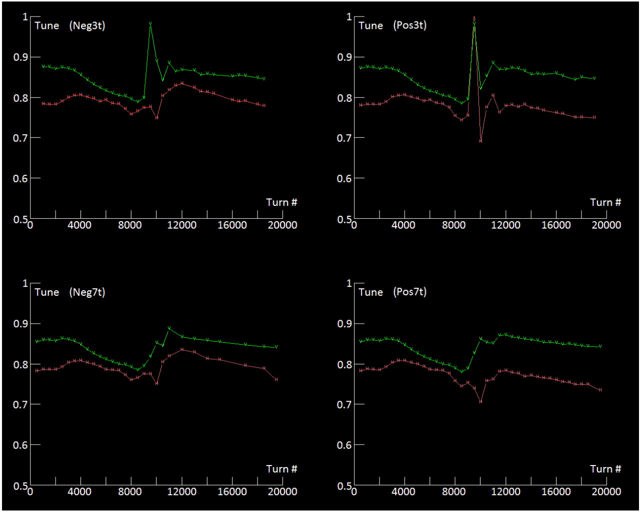


Figure: Measured betatron tunes for 3 and 7 turns. Red and green curves correspond to horizontal and vertical degrees of freedom.

All further results will be coded according to a rule CHROMnntmmc

CHROM variable represent a chromaticity setting (Pos or Neg for positive and negative respectively).

nn stays for the number of turns injected (3, 7 or 14)

Data collection has been performed at 4 different time spots in a Booster cycle:

- $mm = 1$ is a beginning of the cycle (2.84 msec)
- $mm = 2$ is right before transition (16.40 msec)
- $mm = 3$ is right after transition (18.63 msec)
- $mm = 4$ is an end of the cycle (29.51 msec)

3.3 Results and Discussion

In order to extract head-tail modes, one can construct the following matrix consisting of difference signals obtained for different bunches and N turns

$$\begin{bmatrix} \Delta^{(1)}(1) \\ \Delta^{(1)}(2) \\ \dots \\ \Delta^{(1)}(N) \\ \dots \\ \Delta^{(84)}(1) \\ \Delta^{(84)}(2) \\ \dots \\ \Delta^{(84)}(N) \end{bmatrix}$$

Performing the singular value decomposition (SVD) one will obtain spatial and time modes along with corresponding singular values showing the amplitudes.

Spatial mode represents an intra-bunch structure, while time mode shows its behavior over turns. Further split of time mode for different bunches with subsequent Fourier allow to see spectrum of corresponding spatial mode (such a spectrums for different bunches should be added to each other to reduce noise).

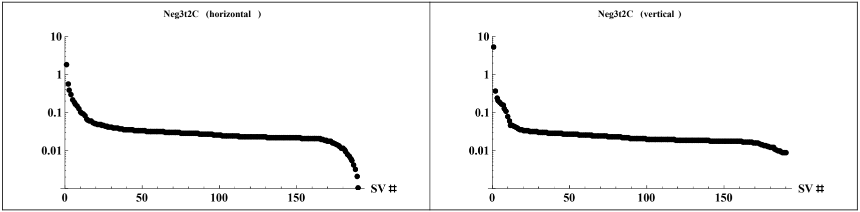
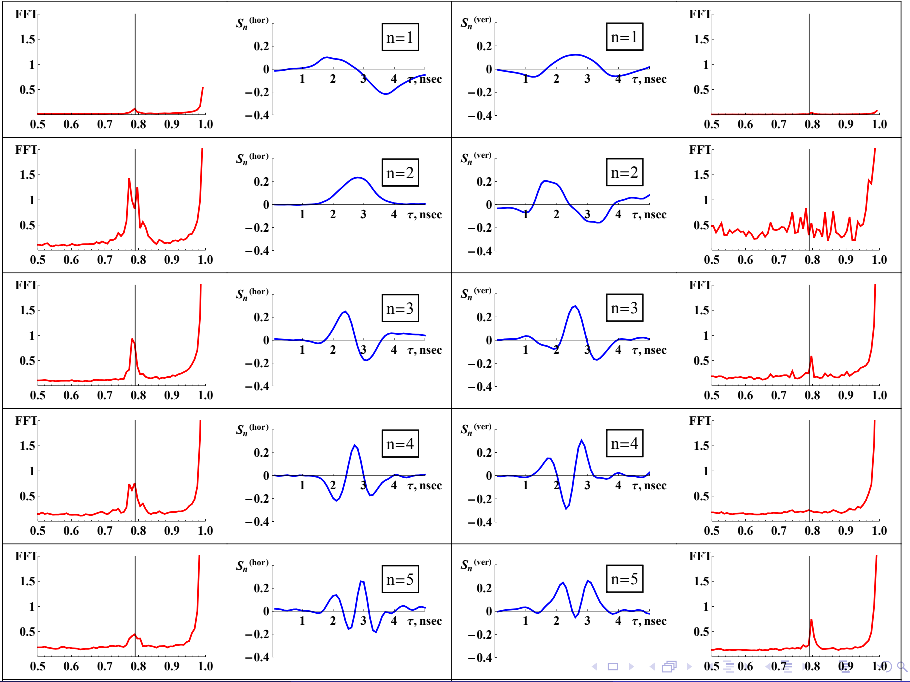
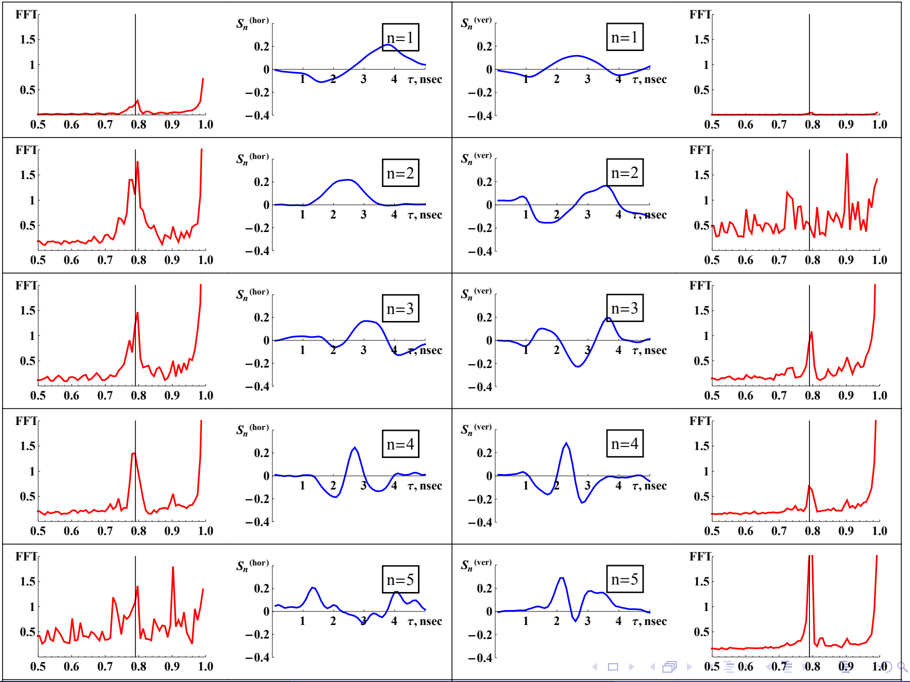
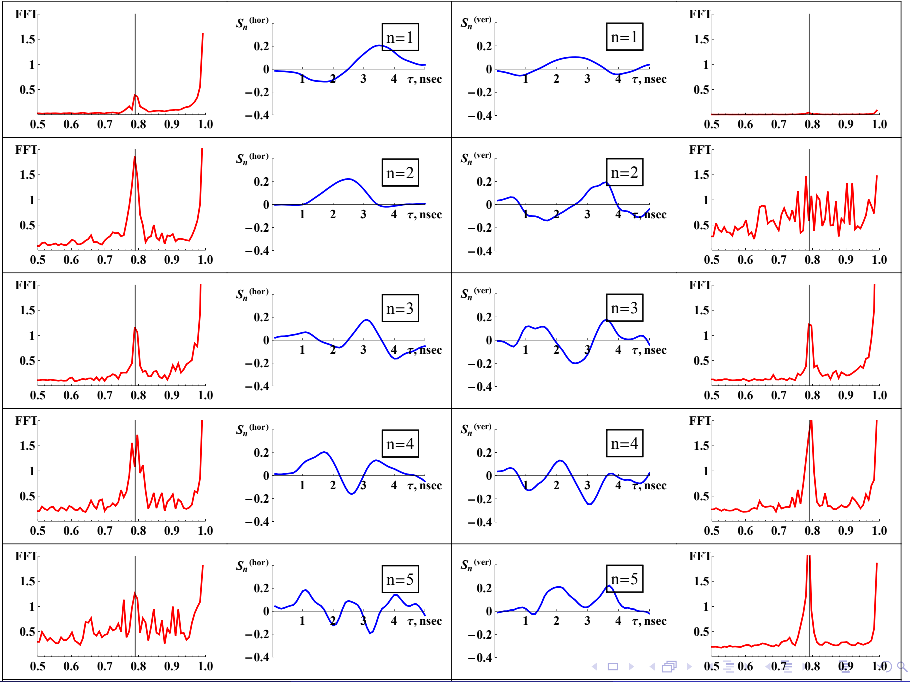


Figure: Singular values for Neg3t2c.


FFT
$S_n^{(hor)}$
n=1
n=2
n=3
n=4
n=5
$S_n^{(ver)}$
τ, nsec

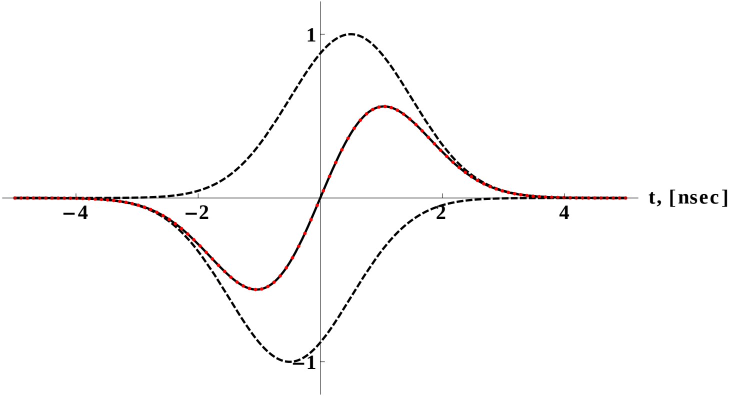

FFT
S_n (hor)
S_n (ver)
τ, nsec
n=1
n=2
n=3
n=4
n=5


FFT
0.5
0.6
0.7
0.8
0.9
1.0
1.5
1
$S_n^{(hor)}$
$S_n^{(ver)}$
0.2
0
−0.2
−0.4
1
2
3
4
τ, nsec
n=1
n=2
n=3
n=4
n=5

Signal Decomposition

Consider a case when the signal from the BPM stripline is overlapped

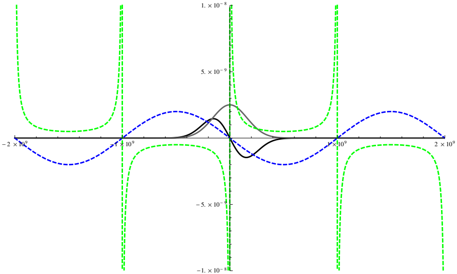
$$V(t) \propto \exp\left[\frac{-(t+\tau)^2}{2\sigma^2}\right] - \exp\left[\frac{-(t-\tau)^2}{2\sigma^2}\right], \qquad \tau < \sigma$$



There are few ways to decompose it:

- **Integration.** If $\tau \ll \sigma$ decomposition is equivalent to $\int_{-\infty}^{\infty} V(t)dt$
- **Fourier space.** If $\tau \sim \sigma$ decomposition can be performed in a Fourier spaceas:

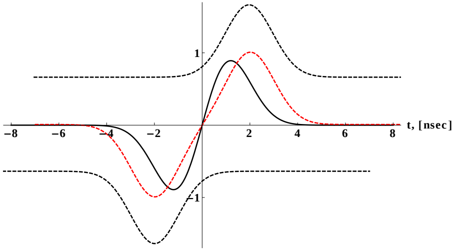
$$\tilde{V}(\omega) \propto -2i\sigma \exp\left[-\omega^2\sigma^2/2\right] \sin[\omega\tau]$$

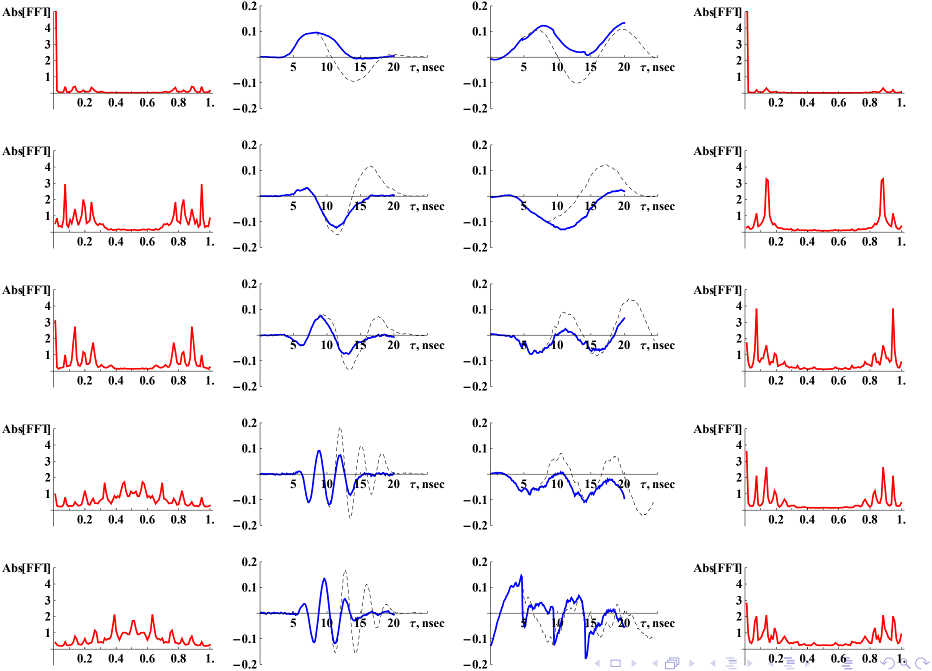


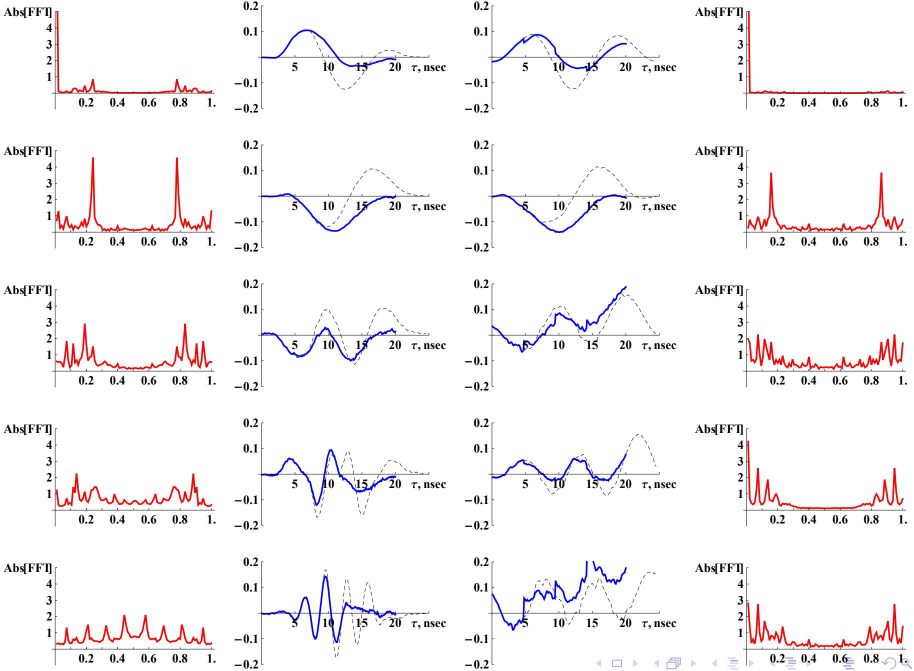
Signal Decomposition

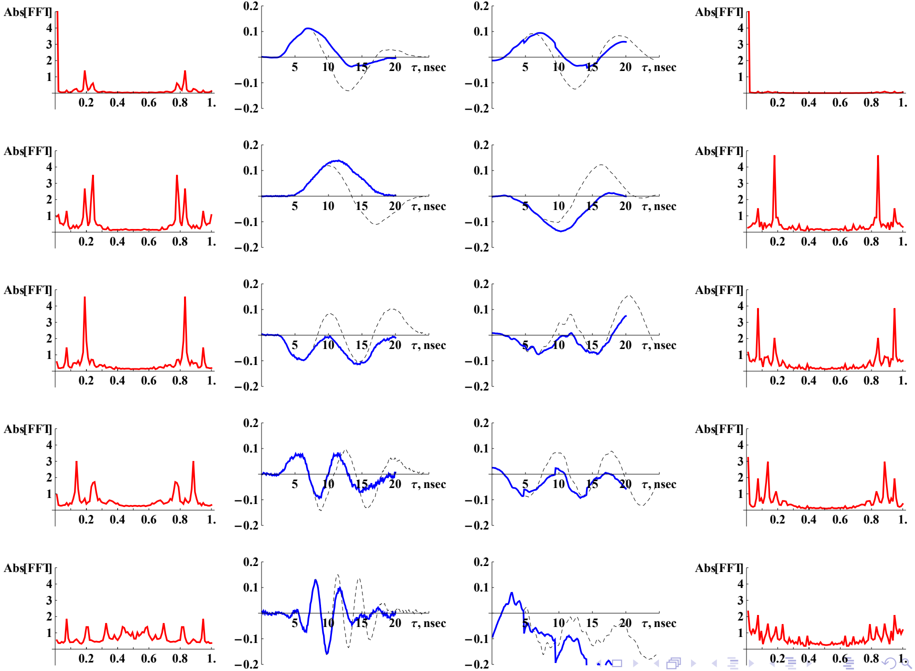
- **Recurrent shift.** One can note that

$$V(t,\tau) + V(t-2\tau,\tau) = V(t,2\tau)$$




Abs[FFT]
0.2
0.4
0.6
0.8
1.
τ, nsec
5
10
15
20
0.2
0.1
−0.1
−0.2


Abs[FFT]
0.2 0.4 0.6 0.8 1.
τ, nsec
5 10 15 20
0.2 0.1 −0.1 −0.2


Abs[FFT]
4
3
2
1
0.2
0.4
0.6
0.8
1.
0.2
0.1
−0.1
−0.2
5
10
15
20
τ, nsec

Acknowledgment

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- All our control room people

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Thank you for your attention!
The End.