Computed Tomography of Transverse Phase Space

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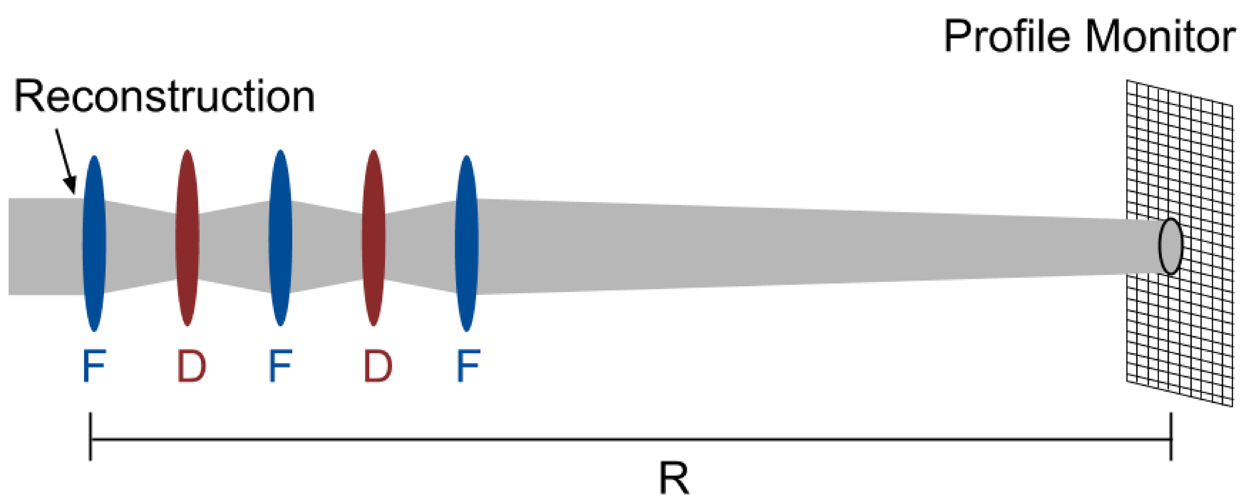
Beam Tomography

The transverse phase space distribution of a particle beam can be reconstructed from profile information with the same computed tomography algorithms used in the medical industry.

Beam profiles are taken at a single point in a beamline while varying the optics between the profile monitor and the reconstruction location.

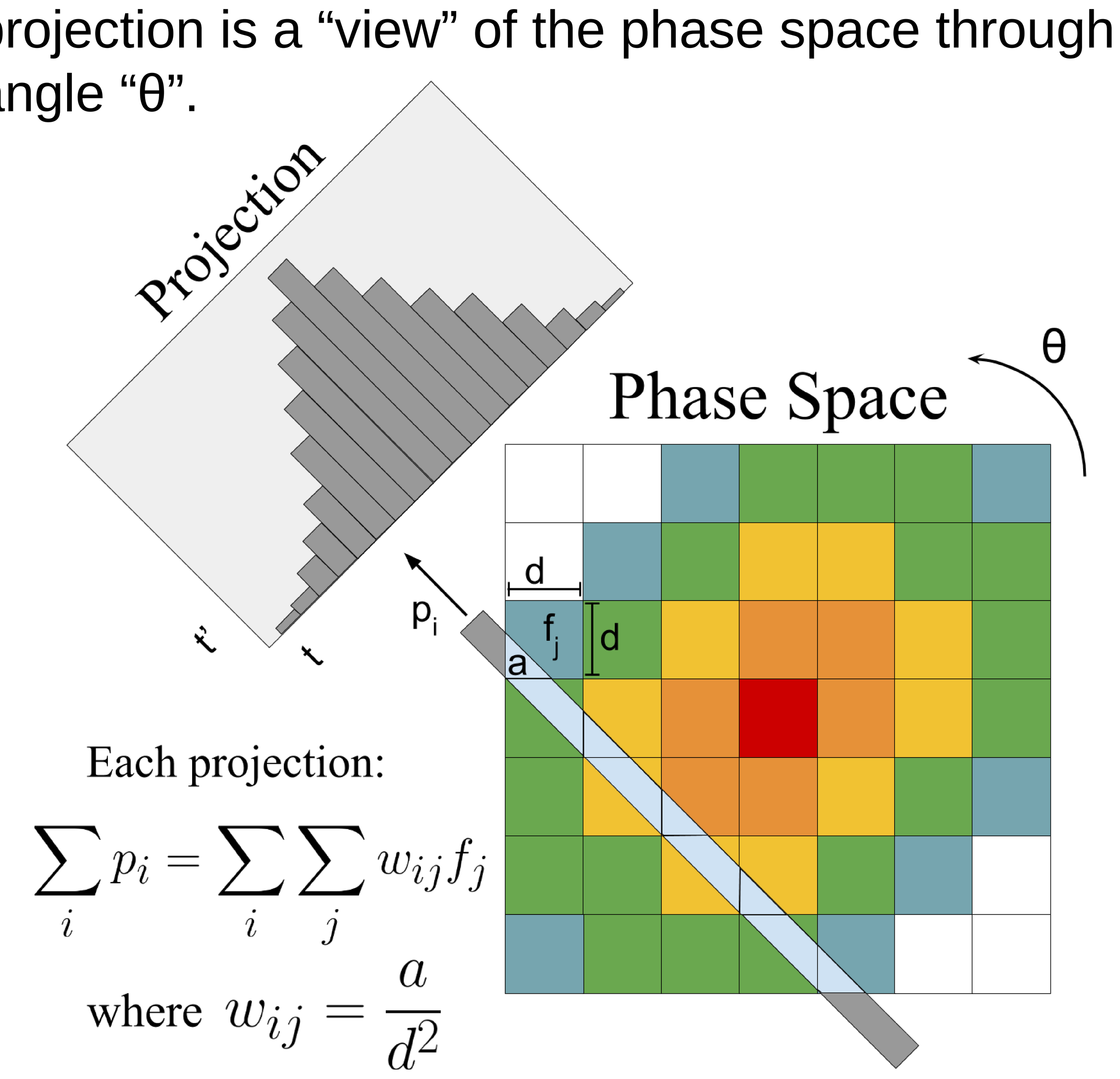


Experimental setup for gathering beam profile data.

For linear transfer matrix R between the point of reconstruction and the profile monitor, we define the scaling factor "s" and phase space orientation angle "θ" as:

$$s = \sqrt{R_{11}^2 + R_{12}^2} \qquad \theta = tan^{-1}\left(\frac{R_{12}}{R_{11}}\right)$$

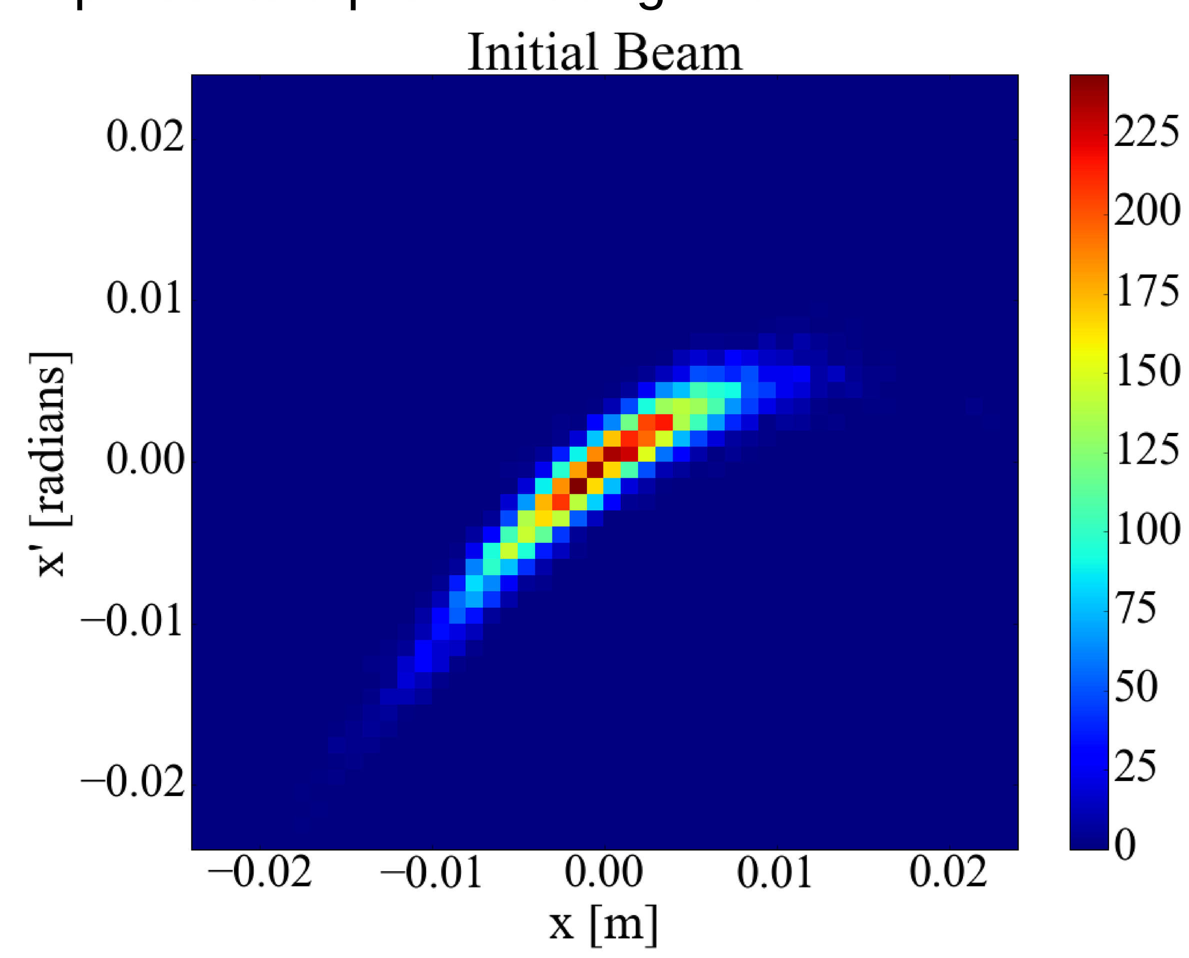
Scaling each beam profile horizontally by "1/s" and vertically by "s" lets us consider each profile as a one-dimensional projection of the two-dimensional phase space for that plane. Each projection is a "view" of the phase space through angle "θ".

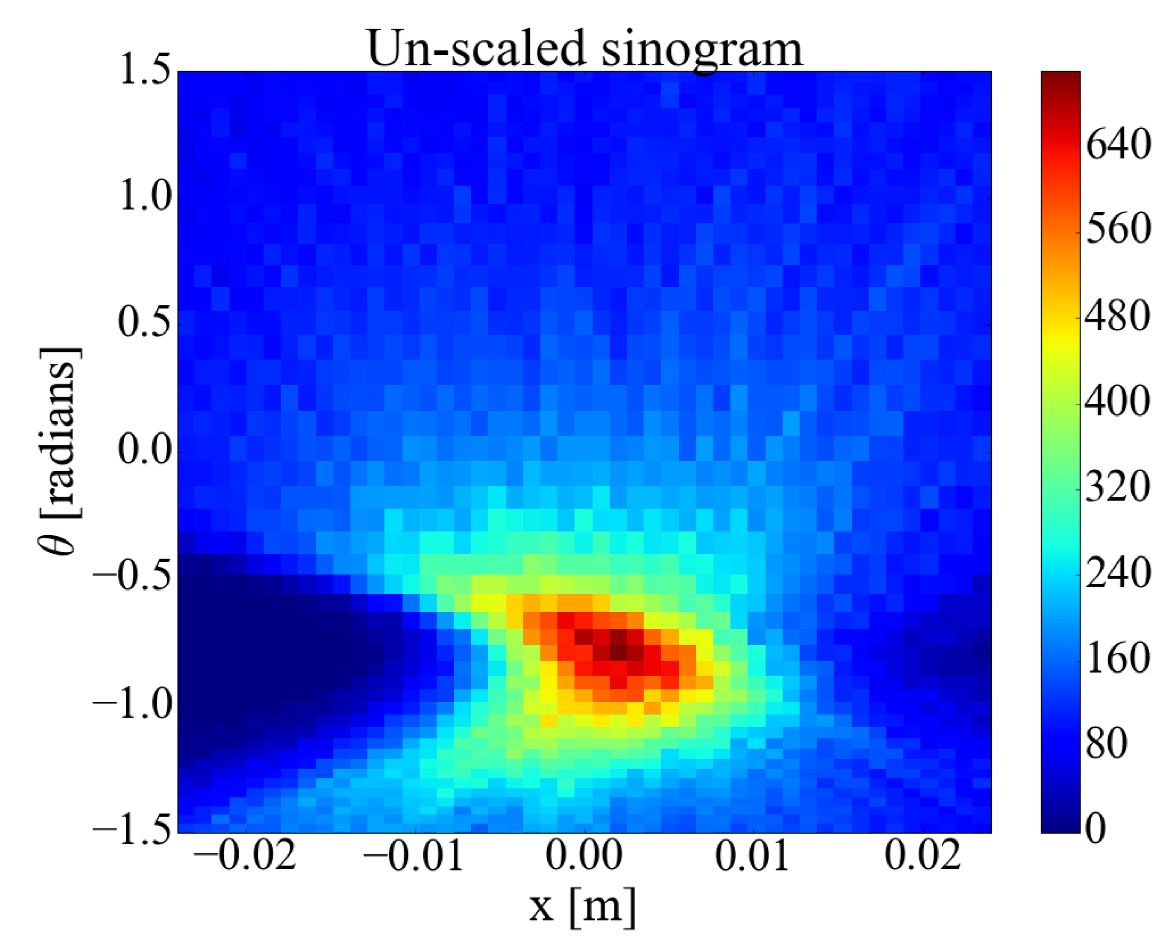


Each scaled beam profile is a one-dimensional projection of the two-dimensional phase space.

Beam Simulation

A deliberately non-elliptical and asymmetric beam distribution is created as a list of particle (x,x') vectors, then passed one-by-one through a symmetric FODO channel using the thick-lens linear transfer matrices. Fixed-bin histograms simulate multiwire profile data for each beamline tune. Each row of the resulting "sinogram" represents a profile at angle "θ".





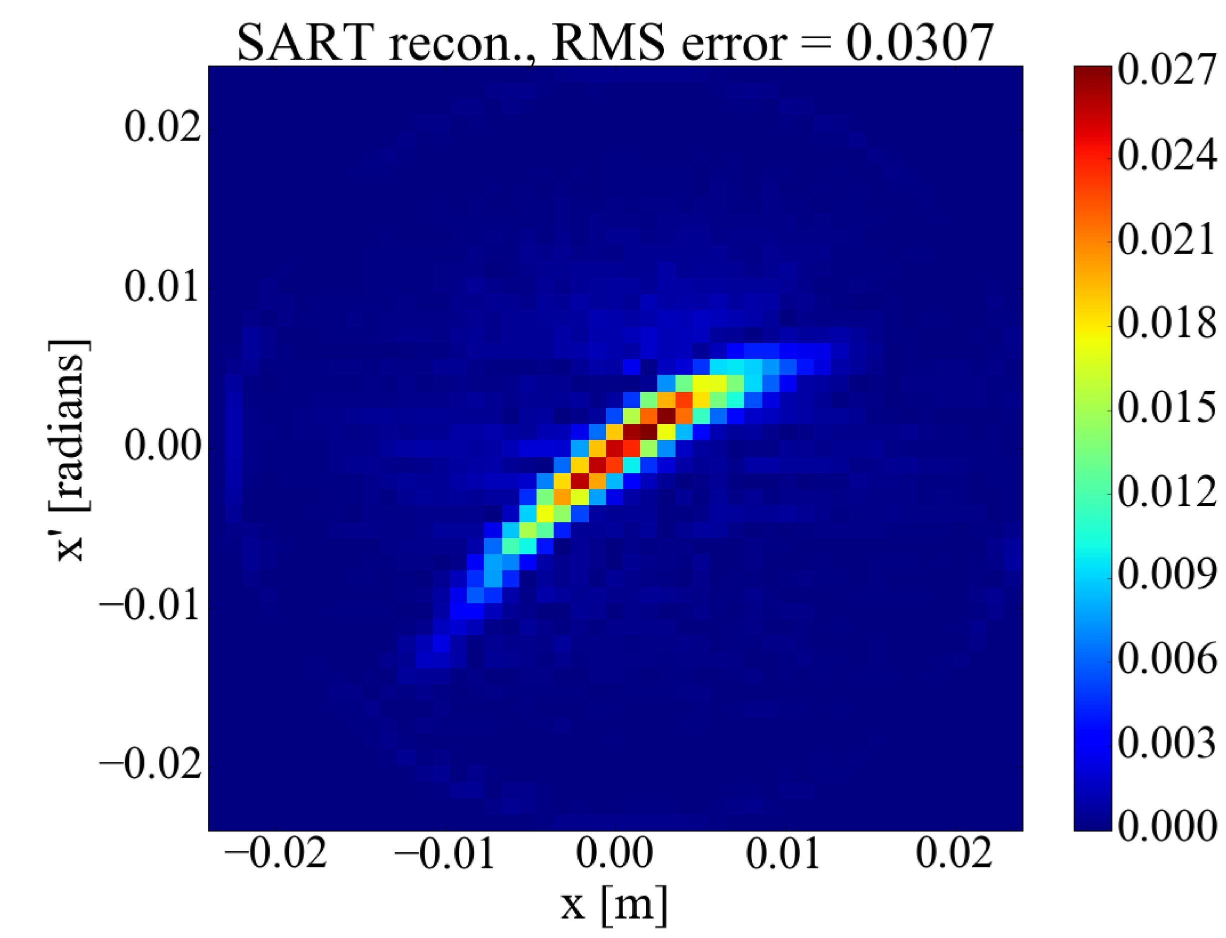
Initial beam distribution and resulting profile data as a function of the phase space orientation angle throughout the simulated scan

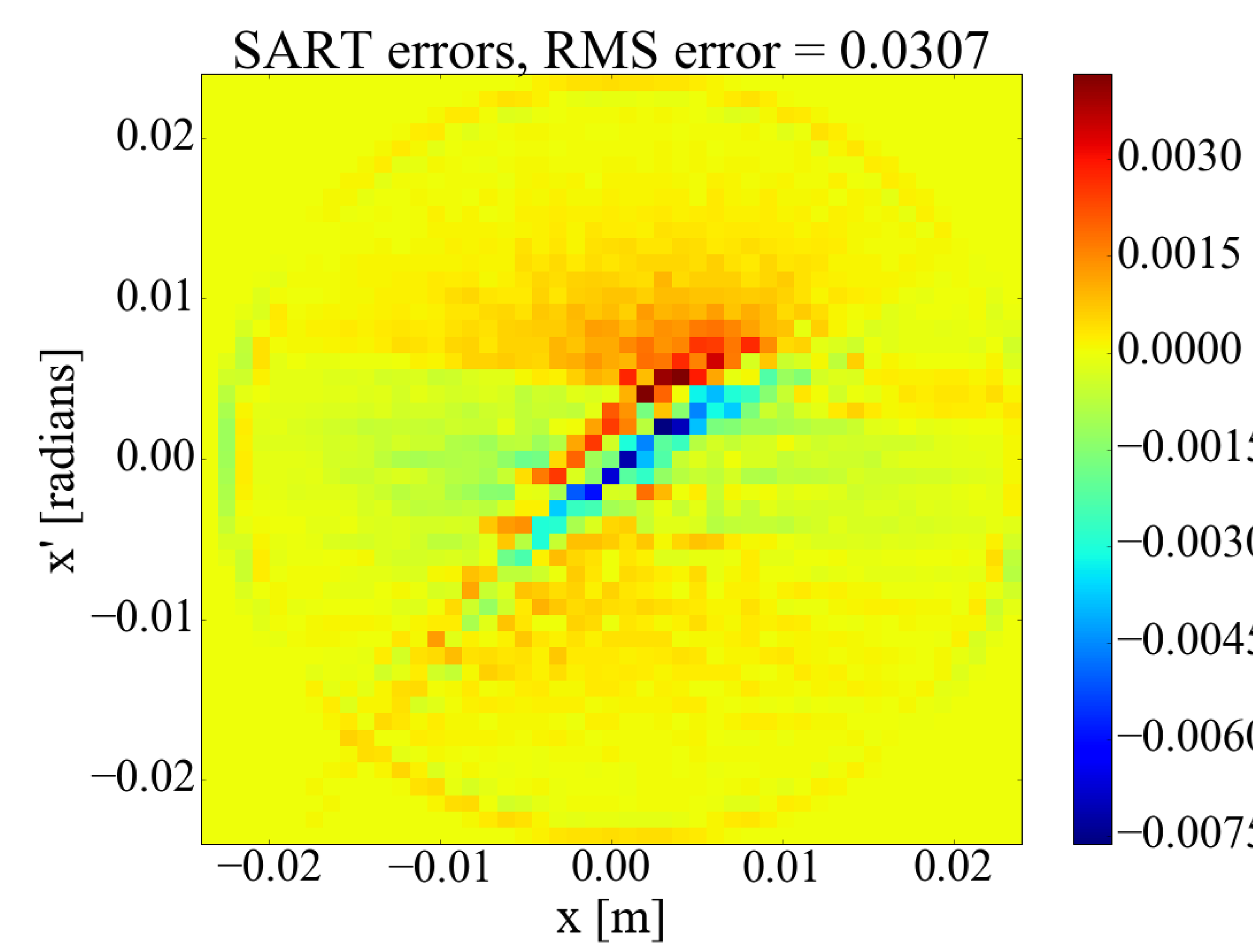
Simultaneous Algebraic Reconstruction

Each imaging ray interacts with a fractional pixel area "a". Thus the following linear system describes each bin "p_i" in a projection, where "f_j" is each pixel's value and "w_{ij}" is typically a non-square matrix of weighting factors based on how the imaging ray interacts with a pixel.

$$p_i = \sum_j w_{ij} f_j$$

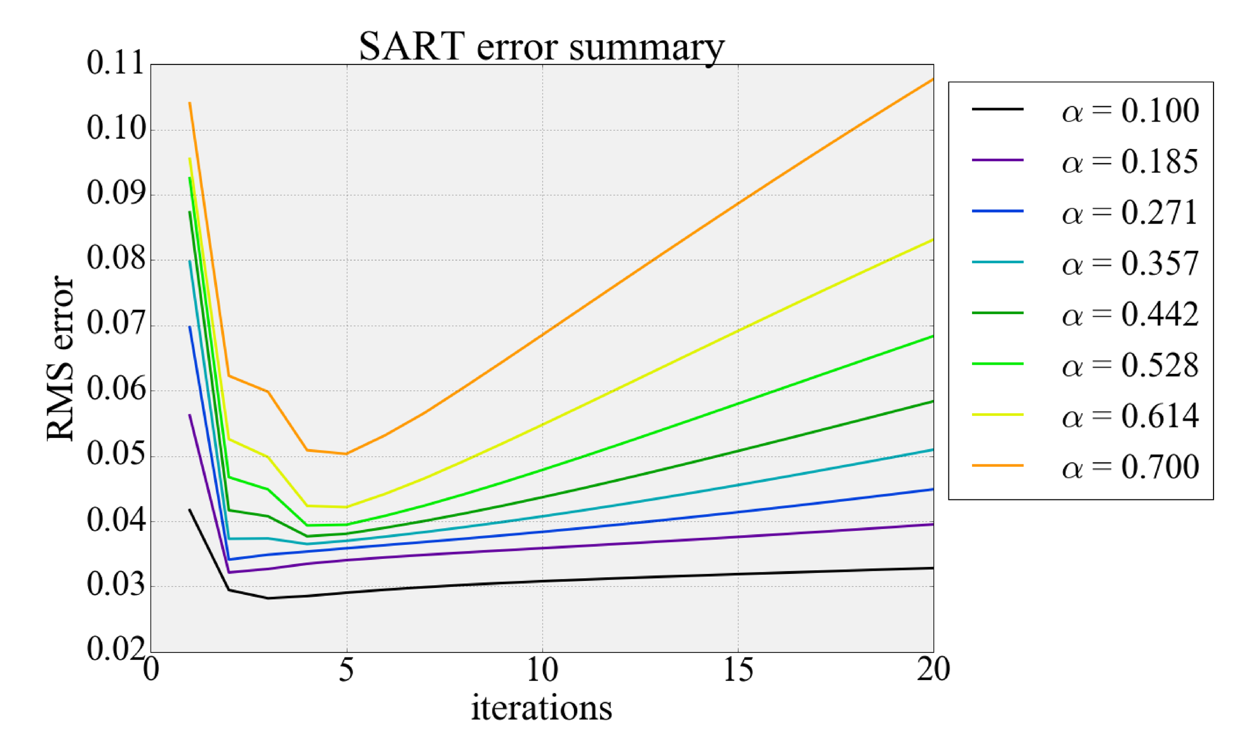
SART is an iterative algorithm that solves this large and under-constrained linear system for "f_j" while comparing solutions to an initial "guess" and applying a correction for each iteration.





SART reconstruction of simulated beam transverse phase space

SART has two free parameters: the relaxation "α" and the number of iterations. Both must be fine-tuned to provide the best reconstruction with lowest artifacts and missing information, i.e. lowest RMS error.



SART reconstructions as a function of algorithm free parameters

References

[1] A.C. Kak and M. Slaney. *Principles of Computerized Tomographic Imaging.* IEEE Press, 1988.

[2] C.B. McKee, P.G. O'Shea, J. M. J. Madey, "Phase Space Tomography of Relativistic Beams", Nucl. Instrum. Methods Phys. Res. A 358,264 (1995)

[3] Scikit-image Python library: *http://scikit-image.org*

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