Multiple Scattering from Thin Targets for the proposed Irradiation Physics Area at Fermilab

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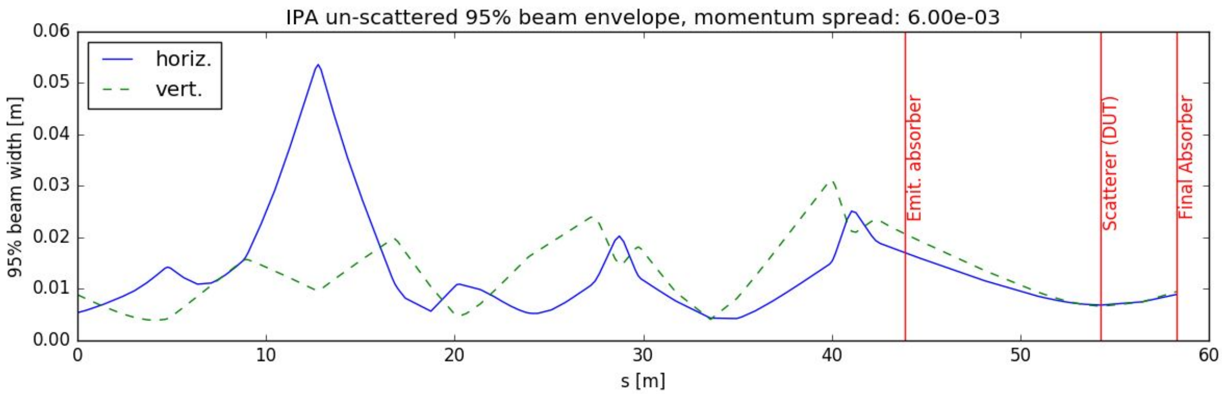


Fig. 1: Irradiation Physics Area beam line from kicker magnet extraction to the material core of the final absorber. [Top] Transverse beta functions showing the approximate positions of the original MuCool Test Area absorber. [Middle] Dispersion functions. [Bottom] 95% physical beam width taking constant 95% emittance of 8 microns. MADX/transport deck can be found at http://cdcvs.fnal.gov/cgi-bin/public-cvs/cvsweb-public.cgi/beamlines/MTA/ Estimated stationing of the front face of the final absorber is 58.25 m. For scattering target placed 4 m before this (3 m of absorber wall, 1 m of shielding wall), the target (DUT) would be at z=54.25 m.

Introduction

This document explores the effect on the primary beam of introducing a thin scattering target 4m upstream of the final absorber in the Irradiation Physics Area. Achieving a 95% (2σ) spot size of 1 cm diameter (half-width $\sigma = 0.25\ cm$) is the subject of a separate document. The passage of the beam through the target will give rise to low-energy secondary particles which are studied elsewhere.

Beam Parameters

At the point of extraction, the kicker magnets, beam projectile's kinetic energy is taken to be 400±0.8 MeV (the 95% window) hydrogen ions, H^-, taking $m_0 = 938$ MeV/c this gives

$\gamma_{rel} = \frac{p}{m_0} + 1 = 1.42$

$\beta_{rel} = \sqrt{1-\gamma^{-2}} = 0.713$. Thus

$p = \beta_{rel}\gamma_{rel}m_0 = 954.1$ MeV/c.

At the front face of the scattering target, the beam profile is nearly circular with 95% containment inside a 0.5 cm radius, and converging. See Table 1 for the second moments of the normally distributed beam width in transverse space and momentum, taken from the proposed tune.

Table 1: Initial second Moments of 95% beam containment

$\sigma_x = 6.87x10^{-3}\ m$	$\sigma_y = 6.62x10^{-3}\ m$
$\sigma_{x'} = 6.62x10^{-3}\ radians$	$\sigma_{y'} = 1.21x10^{-3}\ radians$

Multiple Coulomb Scattering

The irradiated target is expected to multiply-Coulomb-scatter the throughgoing primary beam, such that a particle's final direction is 98% contained ($2.36\,\sigma$) within an angular spread around the original direction (projected into either plane)

$$\theta_0 = \frac{13.6}{\beta c p} z \sqrt{x/X_0}\,[1 + 0.038 \ln(x/X_0)]$$

where β is the particle's velocity in natural units, *p* the momentum in MeV/c, *z* the electric charge, and x/X_0 the material thickness divided by the material's interaction length.

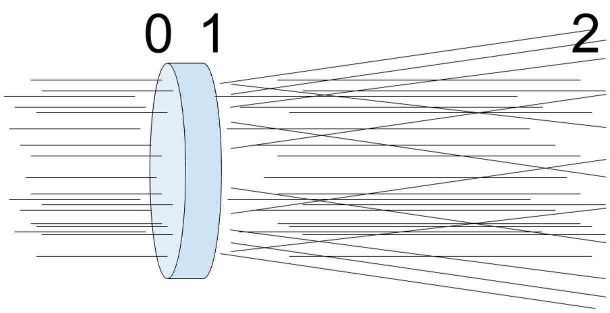
For x/X_0 = 10% and $\beta_{rel} = 0.713,\ p\ = 954.1$ MeV/c, this gives $\theta_{98\%} = 5.77\ mr$

Taking this angle to be normally distributed, the 1-sigma value is $\theta_{rms}\ = 5.77/2.36\ = 2.45\ mr$.

±Nσ	Contained Quantile
1	68%
2	95%
2.36	98%
3	99.7%

For reference, http://pdg.lbl.gov/ lists the radiation length of pure Si as 9.370 cm, so this 10% thickness would be 0.937 cm. This is very thick indeed compared to modern silicon tracking sensors.

Effect on Beamline Optics



We make a thin-target approximation where multiple scattering within the target changes only the trajectories of the particles and not the beam half-widths σ_x and σ_y. **The spot is the same before and after the target.**

The spread of trajectories is increased after the target. This means that after the field-free drift length to the front face of the absorber, **the beam width will be greater at point 2**, and the degree of beam widening depends on the drift length as well as how much scattering takes place in the target. We are interested in the rms beam size σ_2 after this multiple scattering in the target material and a drift of length L.

Adding in quadrature the initial beam rms divergence and θ_{rms} induced by the target, beam emerging from the target will have the rms moment

$$\sigma^2_{x'_1,rms} = (\frac{\sigma_{x'_1 95\%}}{2}) = \sigma^2_{x'_0,rms} + \theta^2_{rms}$$

Doubling σ_{rms} gives the half-width at 95% containment.
In phase space the increase of angular spread of the beam would appear as in Fig. 2

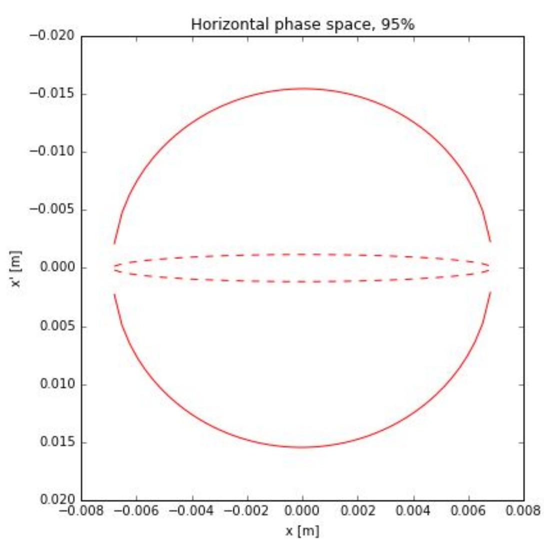


Fig. 2 The beam's 95% containment ellipse expands in the angular (y-axis) direction from dashed line (before target) to solid line (just after target), without change in the spatial direction (x-axis).

Finally, as it drifts approximately 4 meters from the target to the front face of the final absorber, the beam size evolves in a way which depends upon all of the second moments, and on the length L of the drift:

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xx'} \\ \sigma_{xx'} & \sigma_{x'}^2 \end{bmatrix} = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

$$\Sigma_2 = M \Sigma_1 M^T$$

$$\sigma_{x2}^2 = \sigma_{x1}^2 + 2L\sigma_{xx'1} + L^2\sigma_{x'1}^2$$

where the final line is simply identifying first element of the matrix product as σ_{rms} of the beam at the front face of the absorber. The correlation term $\sigma_{xx'}$ is negligible here, and the other two terms we have just calculated. Noting that $\sigma_{95\%} = 2\sigma_{rms}$ the table of second moments can be extended:

Table 1: Second Moments of 95% beam containment

0	$\sigma_{x95\%} = 6.87x10^{-3}\ m$	$\sigma_{y95\%} = 6.62x10^{-3}\ m$
	$\sigma_{x'95\%} = 6.62x10^{-3}\ radians$	$\sigma_{y'95\%} = 1.21x10^{-3}\ radians$
1	$\sigma_{x95\%} = 6.87x10^{-3}\ m$	$\sigma_{y95\%} = 6.62x10^{-3}\ m$
	$\sigma_{x'95\%} = 1.16x10^{-2}\ radians$	$\sigma_{y'95\%} = 1.16x10^{-2}\ radians$
2	$\sigma_{x95\%} = 4.69x10^{-2}\ m$	$\sigma_{y95\%} = 4.69x10^{-2}\ m$

Thus the full width of the scattered primary beam at the front face of the final absorber will be 95% contained within a diameter of 9.38 cm (3.69 inches).

95% Final Beam Diameter for a few choices of target thickness in interaction lengths:

x/X_0	95% containment beam diameter (m)
0.1	0.094
1	0.320
2	0.464
5	0.759
10	1.100