PIP-II RF Studies

Frequency, Bias and Anode Curves for RF Cavity Testing at 20 Hz

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Abstract:

During PIP-II era the Fermilab Booster repetition rate will be increased from 15Hz to 20 Hz. As a first step, the Booster RF system needs to be tested to identify any issues with 20 Hz operation. Therefore, the RF group of AD has under taken a feasibility test by disconnecting two RF cavities (the RF cavities 21 and 22, in the Booster tunnel) from the rest of the twenty RF cavities and attaching a dedicated control that can operate at 20 Hz. For these tests, RF frequency, bias and anode curves as a function of time are needed. In this report we present method used in generating the necessary curves. At the end we also attach the MATLAB program used to generate frequency and bias curves. RF anode curve is generated by performing full ESME calculation from injection to extraction (We present a separate document for ESME simulations).

Introduction

Currently, PIP-II [1] related efforts are underway at Fermilab. During PIP-II era the H^- beam from the newly built SC LINAC will be injected into the Booster and beam will be accelerated to 8 GeV and transferred to the Recycler Ring. Thus, Booster will continue to play a critical role for a few more decades.

Booster is one of the oldest rapid cycling synchrotrons [2] in the world with its dipole magnets ramp on a sinusoidal ramp. The repetition rate of the magnets of Booster is 15Hz. Since 2010 all of the Booster subsystems have been upgraded to provide beam at 15Hz rate as a part of PIP [3]. The PIP-II design goal is to increase the magnet ramp rate from 15 Hz to 20 Hz. This implies that we need to test and upgrade every system 20 Hz compatible. Testing 20 Hz compatibility of Booster RF system is one of the several. The RF Department of the Accelerator

Division [4] has undertaken to test two of the twenty-two RF cavities, 21 and 22. The 20 Hz tests were planned during 2019 July to October long accelerator shutdown.

Important inputs for the cavity testing are 1) RF frequency curve, 2) tuner bias curve and 3) anode curve as a function beam cycle time. In this report we describe the method used to generate these curves.

RF Curves: Frequency and Bias curves

The RF frequency curve is generated using a calculated momentum curve assuming 800 MeV to 8 GeV acceleration and 20 Hz sinusoidal magnetic ramp. The time dependence of momentum of proton as a function of initial and final momenta, P_i and P_f is given by,

$$P(t) = \left(\frac{P_i + P_f}{2}\right) + \left(\frac{P_i - P_f}{2}\right)\cos(2\pi f_B t) \qquad (1)$$

where, f_B =20 Hz is the frequency of the magnetic ramp. The RF frequency f_{RF}, is given by,

$$f_{RF}(t) = h\frac{\beta(t)c}{2\pi R} = \frac{\left[P(t)c/\sqrt{P(t)^2c^2 + m_p^2c^4}\right]c}{2\pi R} \qquad (2)$$

with proton mass m_p and c velocity of light. R =75.47 m is the radius of the Booster. $h = 84$ is the harmonic number of Booster. P_i =1.4632 GeV/c and P_i =8.8889 GeV/c.

RF cavity frequency is varied by using transversely biased ferrite tuners in Booster. One uses frequency dependence of permeability of tuner material (that can be changed by applied bias voltage). We use the operationally tuned bias curve from 400 MeV to 8 GeV case for 15 Hz to generate the one for 800 MeV to 8 GeV at 20 Hz. Figure 1 depicts the operational frequency and bias curves. Note that the frequency curve spans data only up to 35 ms, which is the end of the beam cycle, also it has a 210 μs flat bottom and ~3 ms long flat top. Nevertheless, there is one to one correspondence between bias voltage and RF frequency. We take the nonlinear relation between bias voltage and the RF frequency in the frequency range of 37.94 MHz to 52.81 MHz from operation and a fit a forth order polynomial. The result is shown in Fig. 2. Subsequently, we use this polynomial relationship to the frequency curve obtained for 800 MeV to 8 GeV case for 20 Hz rate. The final result is shown in Fig. 3.

The MATLAB script file used for above calculation is given in Appendix -1.

RF Curves: Anode Curve

The anode curve for the RF cavity test is generated from ESME simulations of beam acceleration from 800 MeV to 8 GeV at 20 Hz rate. At this time, we have performed the single particle beam dynamics simulations without any space charge effect and Booster impedances effects. The baseline specifications for the beam injection [1] assume nearly 550 μs injection time and constant energy front porch (the ref. 1 does not explicitly mention “550 μs front porch”, however, it was implicitly assumed [5]). Figure 4 presents the required RF voltage as a function

of acceleration time from the simulation. The other details of the simulations are presented in a separate document [6].

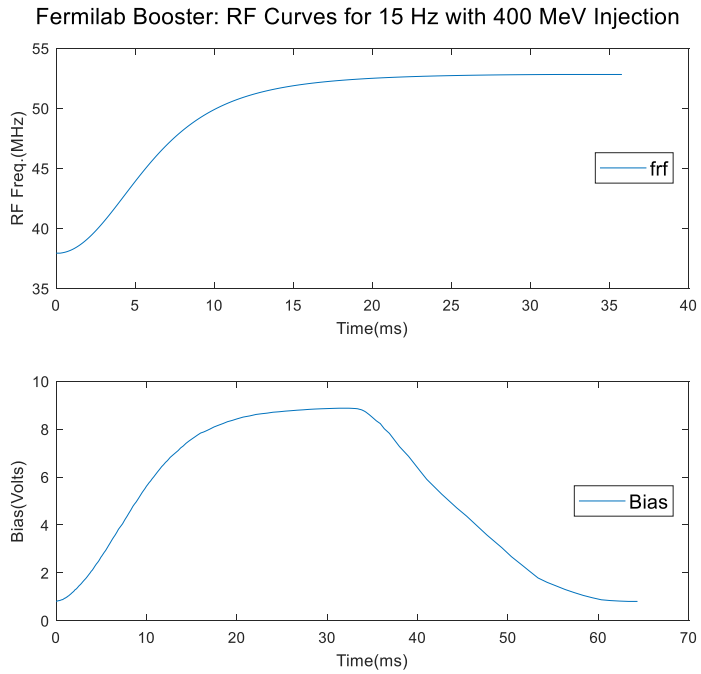

Figure 1: RF frequency and operationally used bias curves for 400 MeV to 8 GeV acceleration case.

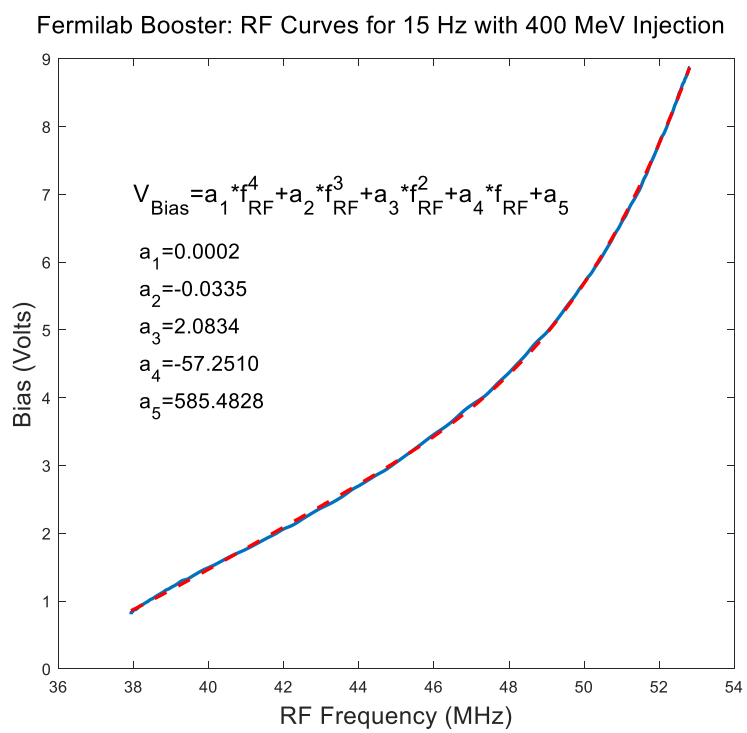

Figure 2: Non-linear relation between operational bias and RF frequency (blue line) and polynomial fit (dashed line).

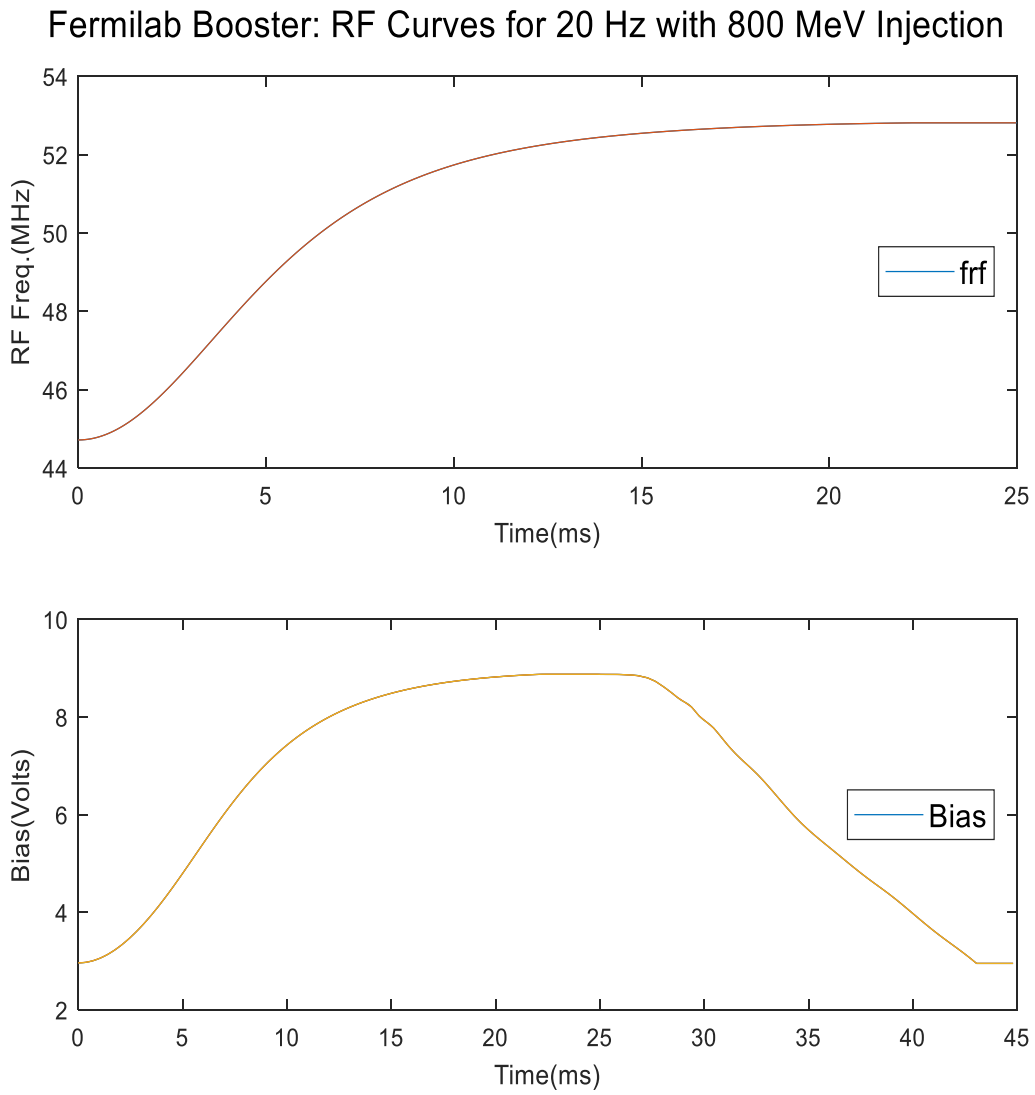

Figure 3: RF frequency and bias curves for 800 MeV to 8 GeV acceleration case with 20 Hz repetition.

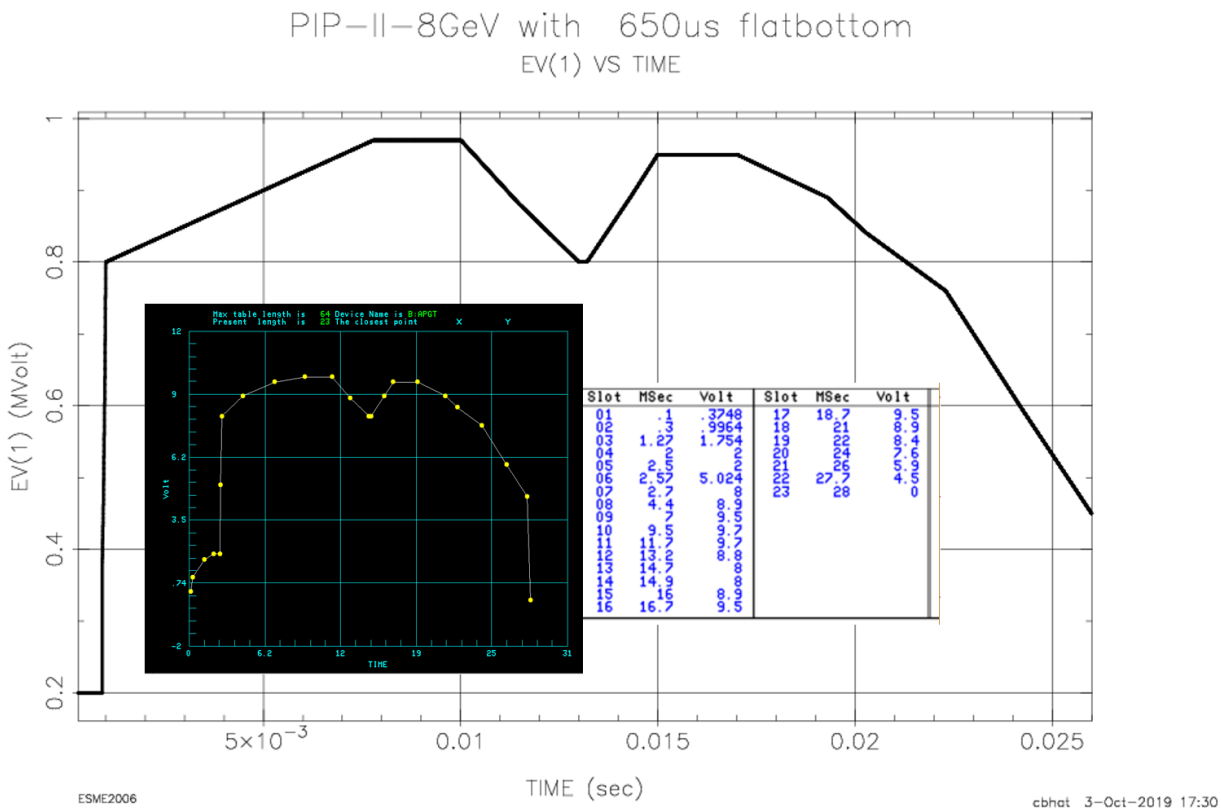

Figure 4: RF anode curve from ESME simulations for 20Hz beam cycle. The insets are the curves used in the RF cavity tests [4].

All of the three curves are used in the RF cavity test at 20 Hz.

References

1. "The PIP-II Conceptual Design Report" V0.03 April 15, 2018.
2. "Booster Synchrotron", Booster Staff, Editor E. L. Hubbard, TM-405-0300, 1973.
3. "Proton Source Improvement Plan" R. Webber et. al, Beams Document 3781-v2 (2011); "FNAL - The Proton Improvement Plan (PIP)" W. Pellico, et al., IPAC2014 (2014) p 3409.
4. V. Grzelak, RF Group, Accelerator Division (private communication).
5. B. E. Chase and P. Derwent (private communications, 2019).
6. "ESME Simulation for 20 Hz Beam Acceleration in Booster during PIP-II era" C. M. Bhat (under preparation, 2019).

Appendix -I: MATLAB program to calculate bias and frequency curve for Booster

```
function []=FB_PIPIIRFfreq-BiasCurve()
%
%   To run  >> t2b.m

%
=====================================================================
%  Following functions are needed
%  mystyle.m
%  nlinfit.m  : Nonlinear fitting routine%
=====================================================================
% 'o' Circle, '+' Plus sign
% '*' Asterisk
% '.' Point
% 'x' Cross
% 'square' or 's'  Square
% 'diamond' or 'd' Diamond
% '^'  Upward-pointing triangle
% 'v' Downward-pointing triangle
% '>' Right-pointing triangle
% '<' Left-pointing triangle
% 'pentagram' or 'p'  Five-pointed star (pentagram)
% 'hexagram' or 'h' Six-pointed star (hexagram)
% % vect_color={'k','g','r','r','m','c','b'};

%


close all; clear all

l_cl={'k','b','g','r','m','c'}
symb={'o','d','x','s','v','^','>','<','p','.','*'}

%  Booster parameters and Other constants
KE_Inj=401; KE_Exit=8000 ;% Energies in MeV
Bramp=15 ;% Hz
mp= 938.272;
R=75.47;
h=84;
c=299792500% m/sec


% ==============================================
% ====================CSV Data=========================
```

```
%     Description of some parameters
% ==============================================

fid='BiasandFreqCurvesFor15HzOpert.csv'

%Read csv files
Titlename = 'Fermilab Booster: RF Curves for 15 Hz with 400 MeV Injection'
% fid='Example.csv'
delimiterIn = ',';% ',' seperated data
headerlinesIn = 6;%  Ignore 1st header line
A = importdata(fid,delimiterIn,headerlinesIn);
tfr15=A.data(:,1);
frq15=A.data(:,2);


tt=A.data(:,3);tbis15=tt(1:201);clear tt;
tt=A.data(:,4);bias15=tt(1:201);clear tt;
mystyle;
subplot(2,1,1); plot(tfr15,frq15),
ylabel('RF Freq.(MHz)');xlabel('Time(ms)')
lgd=legend('frf','Location', 'East')
lgd.FontSize = 13;
subplot(2,1,2); plot(tbis15,bias15),
lgd=legend('Bias','Location', 'East')
lgd.FontSize = 13;
ylabel('Bias(Volts)');xlabel('Time(ms)')
T1=sprintf([Titlename])%'\n','WCM Data FB-20190919-24 ' ])
       annotation('textbox', [0 0.90 1 0.1], ...
       'String', T1,'EdgeColor', 'none', ...
       'HorizontalAlignment', 'center','FontSize',15);
% a=sprintf('%s-pt5nsTo2nsPulses',Titjpg);saveas(gcf,a,'png');

% % Fit Operational Frequency to extract Machine parameters
fun=@(b, t15)h*(((0.5*((sqrt((b(1)+mp).^2-mp.^2))+(sqrt((b(2)+mp).^2-mp.^2))))+...
   (((sqrt((b(1)+mp).^2-mp.^2))-(sqrt((b(2)+mp).^2-mp.^2)))/2)...
   *cos(2*pi*Bramp*t15/1000))./...
   sqrt(((.5*((sqrt((b(1)+mp).^2-mp.^2))+(sqrt((b(2)+mp).^2-mp.^2))))+...
   (((sqrt((b(1)+mp).^2-mp.^2))-(sqrt((b(2)+mp).^2-mp.^2)))/2)*...
   cos(2*pi*Bramp*t15/1000)).^2+mp.^2))*c/(2*pi*b(3))/1e6;

b0=[KE_Inj KE_Exit  R]

% fun=@(b, t15)h*(((0.5*((sqrt((b(1)+mp).^2-mp.^2))+(sqrt((7945.4+mp).^2-mp.^2))))+...
%     (((sqrt((b(1)+mp).^2-mp.^2))-(sqrt((7945.4+mp).^2-mp.^2)))/2)...
%     *cos(2*pi*Bramp*t15/1000))./...
%     sqrt(((.5*((sqrt((b(1)+mp).^2-mp.^2))+(sqrt((7945.4+mp).^2-mp.^2))))+...
```

```
%     (((sqrt((b(1)+mp).^2-mp.^2))-(sqrt((7945.4+mp).^2-mp.^2)))/2)*...
%     cos(2*pi*Bramp*t15/1000)).^2+mp.^2))*c/(2*pi*b(2))/1e6;
% b0=[KE_Inj  R]

tfr15a=tfr15(7:922)
frq15a=frq15(7:922)

bfit = nlinfit(tfr15a,frq15a, fun, b0)
% return
frqfit=fun(bfit,tfr15);
chisq=sum((frqfit-frq15).^2)

R=bfit(3)
Pi=sqrt((bfit(1)+mp).^2-mp.^2)
Pf=sqrt((bfit(2)+mp).^2-mp.^2)
bfit(1)
bfit(2)

bfit = nlinfit(tfr15,frq15, fun, b0)
frqfit=fun(bfit,tfr15);
chisq=sum((frqfit-frq15).^2)



mystyle; plot(tfr15,frq15,tfr15,frqfit)
tbiasnew=(0:0.035:floor(max(tbis15)/0.035)*0.035) ;
bias15new=interp1(tbis15,bias15,tbiasnew,'spline');

mystyle; plot(tbis15,bias15,tbiasnew,bias15new)
max(bias15new)
indx_max = find(bias15new == max(bias15new(:)))
% return
mystyle;
frq15A=frq15(1:902);
bias15newA=bias15new(1:length(frq15A));
plot(frq15A,bias15newA,'Linewidth',2)

fun1=@(bb, f)bb(1)*f.^4+bb(2)*f.^3+bb(3)*f.^2+bb(4)*f+bb(5)
bb0=[0.0002 0.0035 -0.4513 19.439 -280.43]
f=frq15A;
bbfit = nlinfit(f,bias15newA, fun1, bb0)
fitbias=fun1(bbfit,f);
hold on; plot(f,fitbias,'r--','Linewidth',2)
ylabel('Bias (Volts)','FontSize',15);
xlabel('RF Frequency (MHz)','FontSize',15)
T1=sprintf([Titlename])%'\n','WCM Data FB-20190919-24 ' ])
```

```
        annotation('textbox', [0 0.90 1 0.1], ...
        'String', T1,'EdgeColor', 'none', ...
        'HorizontalAlignment', 'center','FontSize',15);
text(38,7,'V_{Bias}=a_1*f_{RF}^4+a_2*f_{RF}^3+a_3*f_{RF}^2+a_4*f_{RF}+a_5','FontSiz
e',15)
text(38,5,sprintf(' a_1=%1.4f \n a_2=%1.4f \n a_3=%1.4f \n a_4=%1.4f \n a_5=%1.4f',...
    bbfit(1),bbfit(2),bbfit(3),bbfit(4),bbfit(5)),'FontSize',13)%
% return

Titlename = 'Fermilab Booster: Bias for PIP-II Operation'
% Generate momentum curve for PIP-II era
KE_Inj=800; KE_Exit=8000 ;% Energies in MeV
Bramp=20 ;% Hz

Es_Inj=KE_Inj+mp; Es_Exit=KE_Exit+mp;% MeV
pInj=sqrt(Es_Inj.^2-mp.^2)
pExt=sqrt(Es_Exit.^2-mp.^2)
dp=(pInj-pExt)/2;
pave=0.5*(pInj+pExt);
tp20=(0:0.035:floor(1000/(Bramp)/2));
p20=pave+dp*cos(2*pi*Bramp*tp20/1000);
beta20=p20/sqrt(p20.^2+mp.^2);
frq20=h*(p20./sqrt(p20.^2+mp.^2))*c/(2*pi*R)/1e6;

for j=1:length(frq20)
    if frq20(j) > 52.811184;
       frq20(j)=52.811184;
    end
end

bias20=fun1(bbfit,frq20);
mystyle;plot(tp20,bias20)
Titlename = 'Fermilab Booster: RF Curves for 20 Hz with 800 MeV Injection'

max(bias15new)
indx_max = find(bias15new == max(bias15new(:)))
k=0;
for j=indx_max:length(bias15new)

if k==0;ji=j;
    if bias15new(j)<bias20(1); k=1;end
end
end
bia=bias15new(indx_max:ji);

bias=[bias20 bia];bias20=bias;tpb20=(0:0.035:(length(bias20)-1)*0.035);
```

```
mystyle;
subplot(2,1,1); plot(tp20,frq20);legend('frf','Location', 'Best')
ylabel('RF Freq.(MHz)');xlabel('Time(ms)')
subplot(2,1,2); plot(tpb20,bias20);legend('Bias','Location', 'Best')
ylabel('Bias(Volts)');xlabel('Time(ms)')
T1=sprintf([Titlename])%'\n','WCM Data FB-20190919-24 ' ])
        annotation('textbox', [0 0.90 1 0.1], ...
        'String', T1,'EdgeColor', 'none', ...
        'HorizontalAlignment', 'center','FontSize',15);

% 128 points in Bias Curve

bias20(length(bias20)+1) =bias20(length(bias20));
tpb20(length(bias20))=44.8;
tb20_128=(0:.35:44.8);
bias20_128=interp1(tpb20,bias20,tb20_128);
% 256 points in Bias Curve
tb20_256=(0:.175:44.8);
bias20_256=interp1(tpb20,bias20,tb20_256);
% 1024 points in frf
t20_1024=(0:0.025:25.6);
frq20_1024=interp1(tp20,frq20,t20_1024);
p20_1024=interp1(tp20,p20,t20_1024);
mystyle;subplot(2,1,1);
plot(tp20,frq20,t20_1024,frq20_1024);
lgd=legend('frf','Location', 'East')
lgd.FontSize = 13;
ylabel('RF Freq.(MHz)');xlabel('Time(ms)')
subplot(2,1,2)
plot(tpb20,bias20,tb20_128,bias20_128 ,tb20_256,bias20_256)
lgd=legend('Bias','Location', 'East')
lgd.FontSize = 13;
ylabel('Bias(Volts)');xlabel('Time(ms)')
T1=sprintf([Titlename])%'\n','WCM Data FB-20190919-24 ' ])
        annotation('textbox', [0 0.90 1 0.1], ...
        'String', T1,'EdgeColor', 'none', ...
        'HorizontalAlignment', 'center','FontSize',15);

Titjpg='PIP-II-20Hz'
file = sprintf('%s-bias-rffreq.xlsx',Titjpg);
% % a=sprintf('%s-Phase Results',Titjpg)
% % tit1={a};
tit={'t(ms)','p(MeV/c)','t(ms)','freq(Mz)','t(ms)','Bias128','t(ms)','Bias256'}
xlswrite(file,tit,1,'A2:H2');
xlswrite(file,t20_1024',1,'A3');
```

```
xlswrite(file,p20_1024',1,'B3');
xlswrite(file,t20_1024',1,'C3');
xlswrite(file,frq20_1024',1,'D3');
xlswrite(file,tb20_128',1,'E3');
xlswrite(file,bias20_128',1,'F3');
xlswrite(file,tb20_256',1,'G3');
xlswrite(file,bias20_256',1,'H3');
end

%%% ========
%%% ==============
%%% ============== Starting point of Functions  ==================
function [] =mystyle()
% Mystyle for plotting
%     mystyle.m :to help plotting
%     set(gca,'FontSize',20,'FontName','Arial')
%     set(gca,'FontWeight','b');set(gca,'LineWidth',2);
% [1 1 0]        y        yellow
% [1 0 1]        m        magenta
% [0 1 1]        c        cyan
% [1 0 0]        r        red change [.5 0 0]
% [0 1 0]        g        green
% [0 0 1]        b        blue
% [1 1 1]        w        white
% [0 0 0]        k        black
% 'o' Circle
% '+' Plus sign
% '*' Asterisk
% '.' Point
% 'x' Cross
% 'square' or 's'  Square
% 'diamond' or 'd' Diamond
% '^'  Upward-pointing triangle
% 'v' Downward-pointing triangle
% '>' Right-pointing triangle
% '<' Left-pointing triangle
% 'pentagram' or 'p'  Five-pointed star (pentagram)
% 'hexagram' or 'h' Six-pointed star (hexagram)

set(0,'Units','pixels')
figure('Position',[100 100 700 600])
set(gca, 'Box', 'on', 'LineWidth', 2, 'Layer', 'top', ...
   'XMinorTick', 'on', 'YMinorTick', 'on', 'XGrid', 'off', 'YGrid', 'off', ...
   'TickDir', 'in', 'TickLength', [.05 .05], ...
   'FontName', 'Arial', 'Fontsize', 19, 'FontWeight', 'normal', ...
   'FontAngle', 'normal');%'XLim', x1([1,end]),'FontAngle', 'italic'
```

```
end


% NonLinear function fit
function [beta,r,J] = nlinfit(X,y,model,beta0)
%NLINFIT Nonlinear least-squares data fitting by the Gauss-Newton method.
%   NLINFIT(X,Y,FUN,BETA0) estimates the coefficients of a nonlinear
%   function using least squares.  Y is a vector of response (dependent
%   variable) values.  Typically, X is a design matrix of predictor
%   (independent variable) values, with one row for each value in Y.
%   However, X may be any array that FUN is prepared to accept.  FUN is
%   a function that accepts two arguments, a coefficient vector and the
%   array X, and returns a vector of fitted Y values.  BETA0 is a vector
%   containing initial values for the coefficients.
%
%   [BETA,R,J] = NLINFIT(X,Y,FUN,BETA0) returns the fitted coefficients
%   BETA, the residuals R, and the Jacobian J.  You can use these outputs
%   with NLPREDCI to produce error estimates on predictions, and with
%   NLPARCI to produce error estimates on the estimated coefficients.
%
%   Examples
%   --------
%   FUN can be specified using @:
%      nlintool(x, y, @myfun, b0)
%   where MYFUN is a MATLAB function such as:
%      function yhat = myfun(beta, x)
%      b1 = beta(1);
%      b2 = beta(2);
%      yhat = 1 ./ (1 + exp(b1 + b2*x));
%
%   FUN can also be an inline object:
%      fun = inline('1 ./ (1 + exp(b(1) + b(2)*x))', 'b', 'x')
%      nlintool(x, y, fun, b0)
%
%   See also NLPARCI, NLPREDCI, NLINTOOL.

%   Copyright 1993-2002 The MathWorks, Inc.
% $Revision: 2.22 $  $Date: 2002/01/17 21:31:27 $

if (nargin<4), error('NLINFIT requires four arguments.'); end

if min(size(y)) ~= 1
   error('Requires a vector second input argument.');
end
y = y(:);
```

```
if size(X,1) == 1 % turn a row vector into a column vector.
   X = X(:);
end

wasnan = (isnan(y) | any(isnan(X),2));
if (any(wasnan))
   y(wasnan) = [];
   X(wasnan,:) = [];
end
n = length(y);

p = length(beta0);
beta0 = beta0(:);

if any(size(feval(model,beta0,X)) ~= size(y))
   error('FUN should return a column vector of the same length as Y.');
end

J = zeros(n,p);
beta = beta0;
betanew = beta + 1;
maxiter = 100;%maxiter = 5;
iter = 0;
betatol = 1.0E-4;
rtol = 1.0E-4;
sse = 1;
sseold = sse;
seps = sqrt(eps);
zbeta = zeros(size(beta));
s10 = sqrt(10);
eyep = eye(p);
zerosp = zeros(p,1);

while (norm((betanew-beta)./(beta+seps)) > betatol | abs(sseold-sse)/(sse+seps) > rtol) & iter <
maxiter
   if iter > 0,
      beta = betanew;
   end

   iter = iter + 1;
   yfit = feval(model,beta,X);
   r = y - yfit;
   sseold = r'*r;

   for k = 1:p,
      delta = zbeta;
```

```
    if (beta(k) == 0)
      nb = sqrt(norm(beta));
      delta(k) = seps * (nb + (nb==0));
    else
      delta(k) = seps*beta(k);
    end
    yplus = feval(model,beta+delta,X);
    J(:,k) = (yplus - yfit)/delta(k);
  end

  Jplus = [J;(1.0E-2)*eyep];
  rplus = [r;zerosp];

  % Levenberg-Marquardt type adjustment
  % Gauss-Newton step -> J\r
  % LM step -> inv(J'*J+constant*eye(p))*J'*r
  step = Jplus\rplus;

  betanew = beta + step;
  yfitnew = feval(model,betanew,X);
  rnew = y - yfitnew;
  sse = rnew'*rnew;
  iter1 = 0;
  while sse > sseold & iter1 < 12
    step = step/s10;
    betanew = beta + step;
    yfitnew = feval(model,betanew,X);
    rnew = y - yfitnew;
    sse = rnew'*rnew;
    iter1 = iter1 + 1;
  end
end
if iter == maxiter
  disp('NLINFIT did NOT converge. Returning results from last iteration.');
end

end
```

Appendix-II: Operational RF frequency and Bias curves

CURVE Freq		CURVE Bias	
Time(us)	frf(MHz)	Time(sec)	Bias(V)
0	37.9356	0	0.804902
0.035	37.9356	0.003	0.804902
0.07	37.9356	0.005	0.804902
0.105	37.9356	0.008	0.804902
0.14	37.9356	0.21	0.829621
0.175	37.9356	0.411	0.841828
0.21	37.9356	0.613	0.872956
0.245	37.9348	0.62	0.872956
0.28	37.9399	0.627	0.872956
0.315	37.94599	0.634	0.872956
0.35	37.95379	0.651	0.872956
0.385	37.96139	0.668	0.872956
0.42	37.97033	0.685	0.879365
0.455	37.98111	0.754	0.885163
0.49	37.9908	0.822	0.904084
0.525	38.00133	0.891	0.921479
0.56	38.01267	0.913	0.928498
0.595	38.02483	0.934	0.934602
0.63	38.03781	0.956	0.934602
0.665	38.0516	0.977	0.9404
0.7	38.0662	0.997	0.9404
0.735	38.08161	1.018	0.946809
0.77	38.09782	1.033	0.946809
0.805	38.11483	1.047	0.952607
0.84	38.13264	1.062	0.952607
0.875	38.15125	1.082	0.959016
0.91	38.17064	1.102	0.964814
0.945	38.19081	1.122	0.971528
0.98	38.21177	1.141	0.977021
1.015	38.23351	1.161	0.977021
1.05	38.25601	1.18	0.983735
1.085	38.27928	1.201	0.988923
1.12	38.30332	1.221	0.988923
1.155	38.32811	1.242	1.00113
1.19	38.35365	1.285	1.013337
1.225	38.37993	1.327	1.026765
1.26	38.40696	1.37	1.032258
1.295	38.43472	1.407	1.044465
1.33	38.46322	1.445	1.056672

1.365	38.49243	1.482	1.068879
1.4	38.52236	1.579	1.100007
1.435	38.553	1.676	1.13083
1.47	38.58435	1.773	1.167756
1.505	38.61639	1.823	1.179353
1.54	38.64912	1.872	1.198579
1.575	38.68254	1.922	1.215974
1.61	38.71664	1.991	1.240388
1.645	38.7514	2.059	1.271821
1.68	38.78683	2.128	1.302644
1.715	38.82292	2.223	1.320344
1.75	38.85965	2.318	1.357881
1.785	38.89702	2.413	1.399995
1.82	38.93503	2.538	1.455232
1.855	38.97367	2.662	1.498872
1.89	39.01292	2.787	1.547395
1.925	39.05278	2.953	1.623384
1.96	39.09324	3.119	1.694795
1.995	39.1343	3.285	1.760408
2.03	39.17594	3.407	1.817476
2.065	39.21816	3.53	1.878511
2.1	39.26095	3.652	1.939241
2.135	39.30431	3.772	1.996614
2.17	39.34821	3.891	2.064058
2.205	39.39266	4.011	2.110445
2.24	39.43764	4.162	2.19742
2.275	39.48315	4.312	2.285921
2.31	39.52917	4.463	2.369234
2.345	39.57571	4.596	2.429964
2.38	39.62274	4.728	2.499239
2.415	39.67027	4.861	2.582552
2.45	39.71828	4.963	2.651827
2.485	39.76676	5.064	2.703402
2.52	39.8157	5.166	2.76108
2.555	39.8651	5.256	2.812655
2.59	39.91494	5.346	2.863619
2.625	39.96522	5.436	2.906954
2.66	40.01593	5.516	2.951205
2.695	40.06706	5.595	3.002169
2.73	40.11859	5.675	3.052523
2.765	40.17052	5.814	3.141024
2.8	40.22285	5.954	3.227694
2.835	40.27555	6.093	3.31589
2.87	40.32862	6.23	3.403781

2.905	40.38206	6.366	3.483737
2.94	40.43584	6.503	3.556979
2.975	40.48997	6.704	3.68027
3.01	40.54444	6.906	3.81882
3.045	40.59923	7.168	3.950656
3.08	40.65433	7.356	4.045566
3.115	40.70973	7.604	4.212192
3.15	40.76544	7.853	4.359287
3.185	40.82143	8.068	4.496921
3.22	40.87769	8.284	4.628452
3.255	40.93423	8.499	4.774021
3.29	40.99102	8.674	4.868015
3.325	41.04806	8.85	4.951023
3.36	41.10534	9.025	5.051731
3.395	41.16285	9.193	5.152439
3.43	41.22059	9.36	5.252842
3.465	41.27853	9.528	5.354771
3.5	41.33668	9.685	5.438389
3.535	41.39502	9.843	5.521702
3.57	41.45355	10	5.615391
3.605	41.51225	10.152	5.689549
3.64	41.57113	10.303	5.7692
3.675	41.63015	10.455	5.831761
3.71	41.68933	10.637	5.929112
3.745	41.74865	10.818	6.011815
3.78	41.8081	11	6.101842
3.815	41.86768	11.226	6.213536
3.85	41.92737	11.452	6.318822
3.885	41.98717	11.678	6.430211
3.92	42.04706	11.894	6.513524
3.955	42.10704	12.111	6.61057
3.99	42.16711	12.327	6.685033
4.025	42.22724	12.627	6.820226
4.06	42.28744	12.926	6.917577
4.095	42.3477	13.226	7.014623
4.13	42.40801	13.505	7.103124
4.165	42.46836	13.785	7.215734
4.2	42.52874	14.064	7.293554
4.235	42.58914	14.376	7.406469
4.27	42.64957	14.688	7.498327
4.305	42.71	15	7.581335
4.34	42.77043	15.333	7.66831
4.375	42.83086	15.667	7.756201
4.41	42.89128	16	7.837683

4.445	42.95168	16.333	7.881018
4.48	43.01205	16.667	7.93717
4.515	43.07238	17	7.997595
4.55	43.13268	17.488	8.092505
4.585	43.19293	17.976	8.163611
4.62	43.25312	18.464	8.231665
4.655	43.31325	18.976	8.311011
4.69	43.37331	19.488	8.363196
4.725	43.4333	20	8.425757
4.76	43.49321	20.709	8.503577
4.795	43.55304	21.419	8.550879
4.83	43.61277	22.128	8.61283
4.865	43.6724	22.752	8.642737
4.9	43.73192	23.376	8.668677
4.935	43.79134	24	8.705298
4.97	43.85064	24.667	8.724219
5.005	43.90981	25.333	8.750159
5.04	43.96886	26	8.769385
5.075	44.02778	26.667	8.790137
5.11	44.08655	27.333	8.805396
5.145	44.14519	28	8.824927
5.18	44.20367	28.667	8.83866
5.215	44.262	29.333	8.849036
5.25	44.32017	30	8.857581
5.285	44.37818	30.667	8.866431
5.32	44.43602	31.333	8.87345
5.355	44.49368	32	8.87345
5.39	44.55117	32.178	8.87345
5.425	44.60847	32.355	8.87345
5.46	44.66559	32.533	8.871619
5.495	44.72252	32.708	8.868262
5.53	44.77926	32.882	8.864295
5.565	44.83579	33.057	8.861243
5.6	44.89213	33.216	8.85575
5.635	44.94825	33.376	8.849036
5.67	45.00417	33.535	8.831641
5.705	45.05988	33.69	8.817908
5.74	45.11536	33.845	8.800208
5.775	45.17063	34	8.770911
5.81	45.22567	34.159	8.741309
5.845	45.28049	34.318	8.701026
5.88	45.33507	34.477	8.654334
5.915	45.38942	34.651	8.60459
5.95	45.44354	34.826	8.547522

5.985	45.49742	35	8.491675
6.02	45.55105	35.438	8.346106
6.055	45.60444	35.877	8.230139
6.09	45.65759	36.315	8.016821
6.125	45.71048	36.877	7.824255
6.16	45.76313	37.438	7.54685
6.195	45.81552	38	7.269445
6.23	45.86765	39	6.864782
6.265	45.91953	40.379	6.20316
6.3	45.97114	41.012	5.904698
6.335	46.0225	42.667	5.285801
6.37	46.07359	44.333	4.710239
6.405	46.12441	45.42	4.368137
6.44	46.17497	47.667	3.579562
6.475	46.22525	49.333	3.045504
6.51	46.27527	50.383	2.6738
6.545	46.32501	51.667	2.287752
6.58	46.37448	52.333	2.083284
6.615	46.42368	53.333	1.783296
6.65	46.4726	54.333	1.601106
6.685	46.52124	55.333	1.451875
6.72	46.56961	56.333	1.301118
6.755	46.61769	57.333	1.172944
6.79	46.6655	58.333	1.054841
6.825	46.71302	59.333	0.957795
6.86	46.76026	60.333	0.872956
6.895	46.80722	61.333	0.839997
6.93	46.8539	62.333	0.819245
6.965	46.90029	63.333	0.804902
7	46.9464	64.333	0.804902
7.035	46.99223	65	0.804902
7.07	47.03776		
7.105	47.08302		
7.14	47.12798		
7.175	47.17266		
7.21	47.21706		
7.245	47.26117		
7.28	47.30499		
7.315	47.34852		
7.35	47.39177		
7.385	47.43473		
7.42	47.47741		
7.455	47.5198		
7.49	47.5619		

7.525	47.60372
7.56	47.64525
7.595	47.68649
7.63	47.72746
7.665	47.76813
7.7	47.80853
7.735	47.84864
7.77	47.88847
7.805	47.92801
7.84	47.96728
7.875	48.00626
7.91	48.04497
7.945	48.08339
7.98	48.12154
8.015	48.15941
8.05	48.197
8.085	48.23432
8.12	48.27136
8.155	48.30813
8.19	48.34462
8.225	48.38085
8.26	48.4168
8.295	48.45248
8.33	48.48789
8.365	48.52303
8.4	48.55791
8.435	48.59252
8.47	48.62687
8.505	48.66095
8.54	48.69478
8.575	48.72834
8.61	48.76164
8.645	48.79468
8.68	48.82746
8.715	48.85999
8.75	48.89227
8.785	48.92429
8.82	48.95605
8.855	48.98757
8.89	49.01884
8.925	49.04986
8.96	49.08063
8.995	49.11116
9.03	49.14145

9.065	49.17149
9.1	49.20129
9.135	49.23085
9.17	49.26017
9.205	49.28926
9.24	49.31811
9.275	49.34672
9.31	49.37511
9.345	49.40326
9.38	49.43118
9.415	49.45888
9.45	49.48634
9.485	49.51359
9.52	49.54061
9.555	49.5674
9.59	49.59398
9.625	49.62034
9.66	49.64648
9.695	49.6724
9.73	49.69811
9.765	49.7236
9.8	49.74888
9.835	49.77395
9.87	49.79882
9.905	49.82347
9.94	49.84792
9.975	49.87217
10.01	49.89621
10.045	49.92005
10.08	49.94369
10.115	49.96713
10.15	49.99038
10.185	50.01343
10.22	50.03629
10.255	50.05895
10.29	50.08142
10.325	50.1037
10.36	50.1258
10.395	50.1477
10.43	50.16942
10.465	50.19096
10.5	50.21232
10.535	50.23349
10.57	50.25448

10.605	50.2753
10.64	50.29594
10.675	50.3164
10.71	50.33669
10.745	50.35681
10.78	50.37675
10.815	50.39653
10.85	50.41614
10.885	50.43558
10.92	50.45485
10.955	50.47396
10.99	50.49291
11.025	50.51169
11.06	50.53032
11.095	50.54878
11.13	50.56709
11.165	50.58524
11.2	50.60324
11.235	50.62108
11.27	50.63877
11.305	50.65631
11.34	50.6737
11.375	50.69094
11.41	50.70803
11.445	50.72498
11.48	50.74178
11.515	50.75844
11.55	50.77495
11.585	50.79132
11.62	50.80756
11.655	50.82365
11.69	50.83961
11.725	50.85543
11.76	50.87112
11.795	50.88667
11.83	50.90209
11.865	50.91737
11.9	50.93253
11.935	50.94756
11.97	50.96245
12.005	50.97723
12.04	50.99187
12.075	51.00639
12.11	51.02079

12.145	51.03506
12.18	51.04921
12.215	51.06324
12.25	51.07715
12.285	51.09095
12.32	51.10462
12.355	51.11818
12.39	51.13162
12.425	51.14495
12.46	51.15817
12.495	51.17127
12.53	51.18426
12.565	51.19714
12.6	51.20992
12.635	51.22258
12.67	51.23514
12.705	51.24759
12.74	51.25993
12.775	51.27217
12.81	51.28431
12.845	51.29634
12.88	51.30827
12.915	51.3201
12.95	51.33183
12.985	51.34346
13.02	51.355
13.055	51.36643
13.09	51.37777
13.125	51.38902
13.16	51.40017
13.195	51.41122
13.23	51.42219
13.265	51.43306
13.3	51.44384
13.335	51.45453
13.37	51.46513
13.405	51.47564
13.44	51.48606
13.475	51.4964
13.51	51.50665
13.545	51.51681
13.58	51.52689
13.615	51.53688
13.65	51.54679

13.685	51.55662
13.72	51.56637
13.755	51.57603
13.79	51.58562
13.825	51.59513
13.86	51.60455
13.895	51.6139
13.93	51.62317
13.965	51.63237
14	51.64149
14.035	51.65053
14.07	51.6595
14.105	51.66839
14.14	51.67721
14.175	51.68596
14.21	51.69464
14.245	51.70324
14.28	51.71178
14.315	51.72024
14.35	51.72864
14.385	51.73696
14.42	51.74522
14.455	51.75341
14.49	51.76153
14.525	51.76959
14.56	51.77758
14.595	51.78551
14.63	51.79337
14.665	51.80117
14.7	51.8089
14.735	51.81657
14.77	51.82418
14.805	51.83173
14.84	51.83922
14.875	51.84664
14.91	51.85401
14.945	51.86131
14.98	51.86856
15.015	51.87575
15.05	51.88288
15.085	51.88995
15.12	51.89697
15.155	51.90393
15.19	51.91083

15.225	51.91768
15.26	51.92448
15.295	51.93122
15.33	51.9379
15.365	51.94454
15.4	51.95112
15.435	51.95764
15.47	51.96412
15.505	51.97054
15.54	51.97691
15.575	51.98324
15.61	51.98951
15.645	51.99573
15.68	52.0019
15.715	52.00803
15.75	52.0141
15.785	52.02013
15.82	52.02611
15.855	52.03205
15.89	52.03793
15.925	52.04377
15.96	52.04957
15.995	52.05532
16.03	52.06102
16.065	52.06668
16.1	52.0723
16.135	52.07787
16.17	52.0834
16.205	52.08888
16.24	52.09433
16.275	52.09973
16.31	52.10508
16.345	52.1104
16.38	52.11568
16.415	52.12091
16.45	52.1261
16.485	52.13126
16.52	52.13637
16.555	52.14145
16.59	52.14648
16.625	52.15148
16.66	52.15644
16.695	52.16136
16.73	52.16624

16.765	52.17109
16.8	52.17589
16.835	52.18067
16.87	52.1854
16.905	52.1901
16.94	52.19476
16.975	52.19939
17.01	52.20398
17.045	52.20854
17.08	52.21306
17.115	52.21755
17.15	52.222
17.185	52.22643
17.22	52.23081
17.255	52.23517
17.29	52.23949
17.325	52.24378
17.36	52.24803
17.395	52.25226
17.43	52.25645
17.465	52.26061
17.5	52.26474
17.535	52.26884
17.57	52.27291
17.605	52.27695
17.64	52.28096
17.675	52.28494
17.71	52.28888
17.745	52.2928
17.78	52.29669
17.815	52.30056
17.85	52.30439
17.885	52.30819
17.92	52.31197
17.955	52.31572
17.99	52.31944
18.025	52.32313
18.06	52.3268
18.095	52.33043
18.13	52.33405
18.165	52.33763
18.2	52.34119
18.235	52.34472
18.27	52.34823

18.305	52.35171
18.34	52.35517
18.375	52.3586
18.41	52.362
18.445	52.36539
18.48	52.36874
18.515	52.37207
18.55	52.37538
18.585	52.37866
18.62	52.38192
18.655	52.38516
18.69	52.38837
18.725	52.39156
18.76	52.39473
18.795	52.39787
18.83	52.40099
18.865	52.40409
18.9	52.40717
18.935	52.41022
18.97	52.41325
19.005	52.41626
19.04	52.41925
19.075	52.42222
19.11	52.42517
19.145	52.42809
19.18	52.431
19.215	52.43388
19.25	52.43674
19.285	52.43958
19.32	52.44241
19.355	52.44521
19.39	52.44799
19.425	52.45075
19.46	52.4535
19.495	52.45622
19.53	52.45892
19.565	52.46161
19.6	52.46428
19.635	52.46692
19.67	52.46955
19.705	52.47216
19.74	52.47475
19.775	52.47733
19.81	52.47988

19.845	52.48242
19.88	52.48494
19.915	52.48744
19.95	52.48993
19.985	52.49239
20.02	52.49484
20.055	52.49728
20.09	52.49969
20.125	52.50209
20.16	52.50447
20.195	52.50684
20.23	52.50919
20.265	52.51152
20.3	52.51384
20.335	52.51614
20.37	52.51842
20.405	52.52069
20.44	52.52294
20.475	52.52518
20.51	52.5274
20.545	52.52961
20.58	52.5318
20.615	52.53398
20.65	52.53614
20.685	52.53829
20.72	52.54042
20.755	52.54253
20.79	52.54464
20.825	52.54672
20.86	52.5488
20.895	52.55086
20.93	52.5529
20.965	52.55493
21	52.55695
21.035	52.55895
21.07	52.56094
21.105	52.56292
21.14	52.56488
21.175	52.56683
21.21	52.56877
21.245	52.57069
21.28	52.5726
21.315	52.5745
21.35	52.57638

21.385	52.57825
21.42	52.58011
21.455	52.58196
21.49	52.58379
21.525	52.58561
21.56	52.58742
21.595	52.58922
21.63	52.591
21.665	52.59277
21.7	52.59453
21.735	52.59628
21.77	52.59802
21.805	52.59974
21.84	52.60145
21.875	52.60316
21.91	52.60485
21.945	52.60652
21.98	52.60819
22.015	52.60985
22.05	52.61149
22.085	52.61312
22.12	52.61475
22.155	52.61636
22.19	52.61796
22.225	52.61955
22.26	52.62113
22.295	52.6227
22.33	52.62426
22.365	52.62581
22.4	52.62734
22.435	52.62887
22.47	52.63039
22.505	52.63189
22.54	52.63339
22.575	52.63488
22.61	52.63635
22.645	52.63782
22.68	52.63928
22.715	52.64072
22.75	52.64216
22.785	52.64359
22.82	52.64501
22.855	52.64642
22.89	52.64782

22.925	52.64921
22.96	52.65059
22.995	52.65196
23.03	52.65332
23.065	52.65468
23.1	52.65602
23.135	52.65736
23.17	52.65868
23.205	52.66
23.24	52.66131
23.275	52.66261
23.31	52.6639
23.345	52.66518
23.38	52.66646
23.415	52.66772
23.45	52.66898
23.485	52.67023
23.52	52.67147
23.555	52.6727
23.59	52.67392
23.625	52.67514
23.66	52.67635
23.695	52.67755
23.73	52.67874
23.765	52.67992
23.8	52.6811
23.835	52.68227
23.87	52.68343
23.905	52.68458
23.94	52.68572
23.975	52.68686
24.01	52.68799
24.045	52.68911
24.08	52.69022
24.115	52.69133
24.15	52.69243
24.185	52.69352
24.22	52.69461
24.255	52.69568
24.29	52.69675
24.325	52.69782
24.36	52.69887
24.395	52.69992
24.43	52.70096

24.465	52.702
24.5	52.70303
24.535	52.70405
24.57	52.70506
24.605	52.70607
24.64	52.70707
24.675	52.70806
24.71	52.70905
24.745	52.71003
24.78	52.711
24.815	52.71197
24.85	52.71293
24.885	52.71388
24.92	52.71483
24.955	52.71577
24.99	52.7167
25.025	52.71763
25.06	52.71855
25.095	52.71947
25.13	52.72038
25.165	52.72128
25.2	52.72218
25.235	52.72307
25.27	52.72395
25.305	52.72483
25.34	52.72571
25.375	52.72657
25.41	52.72743
25.445	52.72829
25.48	52.72914
25.515	52.72998
25.55	52.73082
25.585	52.73165
25.62	52.73247
25.655	52.73329
25.69	52.73411
25.725	52.73492
25.76	52.73572
25.795	52.73652
25.83	52.73731
25.865	52.73809
25.9	52.73887
25.935	52.73965
25.97	52.74042

26.005	52.74118
26.04	52.74194
26.075	52.7427
26.11	52.74345
26.145	52.74419
26.18	52.74493
26.215	52.74566
26.25	52.74639
26.285	52.74711
26.32	52.74783
26.355	52.74854
26.39	52.74924
26.425	52.74995
26.46	52.75064
26.495	52.75133
26.53	52.75202
26.565	52.7527
26.6	52.75338
26.635	52.75405
26.67	52.75472
26.705	52.75538
26.74	52.75604
26.775	52.75669
26.81	52.75734
26.845	52.75798
26.88	52.75862
26.915	52.75925
26.95	52.75988
26.985	52.76051
27.02	52.76113
27.055	52.76174
27.09	52.76235
27.125	52.76296
27.16	52.76356
27.195	52.76415
27.23	52.76475
27.265	52.76533
27.3	52.76592
27.335	52.7665
27.37	52.76707
27.405	52.76764
27.44	52.76821
27.475	52.76877
27.51	52.76932

27.545	52.76988
27.58	52.77043
27.615	52.77097
27.65	52.77151
27.685	52.77204
27.72	52.77258
27.755	52.7731
27.79	52.77363
27.825	52.77414
27.86	52.77466
27.895	52.77517
27.93	52.77568
27.965	52.77618
28	52.77668
28.035	52.77717
28.07	52.77766
28.105	52.77815
28.14	52.77863
28.175	52.77911
28.21	52.77958
28.245	52.78005
28.28	52.78052
28.315	52.78098
28.35	52.78144
28.385	52.78189
28.42	52.78234
28.455	52.78279
28.49	52.78323
28.525	52.78367
28.56	52.78411
28.595	52.78454
28.63	52.78497
28.665	52.78539
28.7	52.78581
28.735	52.78623
28.77	52.78664
28.805	52.78705
28.84	52.78746
28.875	52.78786
28.91	52.78826
28.945	52.78865
28.98	52.78904
29.015	52.78943
29.05	52.78981

29.085	52.79019
29.12	52.79057
29.155	52.79094
29.19	52.79131
29.225	52.79168
29.26	52.79204
29.295	52.7924
29.33	52.79275
29.365	52.79311
29.4	52.79346
29.435	52.7938
29.47	52.79414
29.505	52.79448
29.54	52.79481
29.575	52.79515
29.61	52.79547
29.645	52.7958
29.68	52.79612
29.715	52.79644
29.75	52.79675
29.785	52.79706
29.82	52.79737
29.855	52.79767
29.89	52.79797
29.925	52.79823
29.96	52.79863
29.995	52.7991
30.03	52.7993
30.065	52.79963
30.1	52.79997
30.135	52.8003
30.17	52.80057
30.205	52.80084
30.24	52.8011
30.275	52.80137
30.31	52.80171
30.345	52.80204
30.38	52.80231
30.415	52.80264
30.45	52.80304
30.485	52.80338
30.52	52.80371
30.555	52.80404
30.59	52.80438

30.625	52.80465
30.66	52.80485
30.695	52.80505
30.73	52.80531
30.765	52.80545
30.8	52.80565
30.835	52.80578
30.87	52.80605
30.905	52.80632
30.94	52.80652
30.975	52.80672
31.01	52.80705
31.045	52.80725
31.08	52.80752
31.115	52.80772
31.15	52.80792
31.185	52.80805
31.22	52.80826
31.255	52.80846
31.29	52.80859
31.325	52.80866
31.36	52.80892
31.395	52.80912
31.43	52.80932
31.465	52.80953
31.5	52.80966
31.535	52.80979
31.57	52.80993
31.605	52.80999
31.64	52.81013
31.675	52.81033
31.71	52.81053
31.745	52.81066
31.78	52.81066
31.815	52.81086
31.85	52.81106
31.885	52.81113
31.92	52.8112
31.955	52.81126
31.99	52.81126
32.025	52.81126
32.06	52.81126
32.095	52.81117
32.13	52.81114

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32.235	52.81123
32.27	52.81118
32.305	52.81118
32.34	52.81118
32.375	52.81118
32.41	52.81118
32.445	52.81118
32.48	52.81118
32.515	52.81118
32.55	52.81118
32.585	52.81118
32.62	52.81118
32.655	52.81118
32.69	52.81118
32.725	52.81118
32.76	52.81118
32.795	52.81118
32.83	52.81118
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32.9	52.81118
32.935	52.81118
32.97	52.81118
33.005	52.81118
33.04	52.81118
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33.11	52.81118
33.145	52.81118
33.18	52.81118
33.215	52.81118
33.25	52.81118
33.285	52.81118
33.32	52.81118
33.355	52.81118
33.39	52.81118
33.425	52.81118
33.46	52.81118
33.495	52.81118
33.53	52.81118
33.565	52.81118
33.6	52.81118
33.635	52.81118
33.67	52.81118

33.705	52.81118
33.74	52.81118
33.775	52.81118
33.81	52.81118
33.845	52.81118
33.88	52.81118
33.915	52.81118
33.95	52.81118
33.985	52.81118
34.02	52.81118
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34.09	52.81118
34.125	52.81118
34.16	52.81118
34.195	52.81118
34.23	52.81118
34.265	52.81118
34.3	52.81118
34.335	52.81118
34.37	52.81118
34.405	52.81118
34.44	52.81118
34.475	52.81118
34.51	52.81118
34.545	52.81118
34.58	52.81118
34.615	52.81118
34.65	52.81118
34.685	52.81118
34.72	52.81118
34.755	52.81118
34.79	52.81118
34.825	52.81118
34.86	52.81118
34.895	52.81118
34.93	52.81118
34.965	52.81118
35	52.81118
35.035	52.81118
35.07	52.81118
35.105	52.81118
35.14	52.81118
35.175	52.81118
35.21	52.81118

35.245	52.81118
35.28	52.81118
35.315	52.81118
35.35	52.81118
35.385	52.81118
35.42	52.81118
35.455	52.81118
35.49	52.81118
35.525	52.81118
35.56	52.81118
35.595	52.81118
35.63	52.81118
35.665	52.81118
35.7	52.81118
35.735	52.81118
35.777	52.81118