One Step Closer to a Muon Collider: Demonstration of Ionization Cooling by MICE

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- Beamstrahlung smears energy resolution even for a linear e+e- collider

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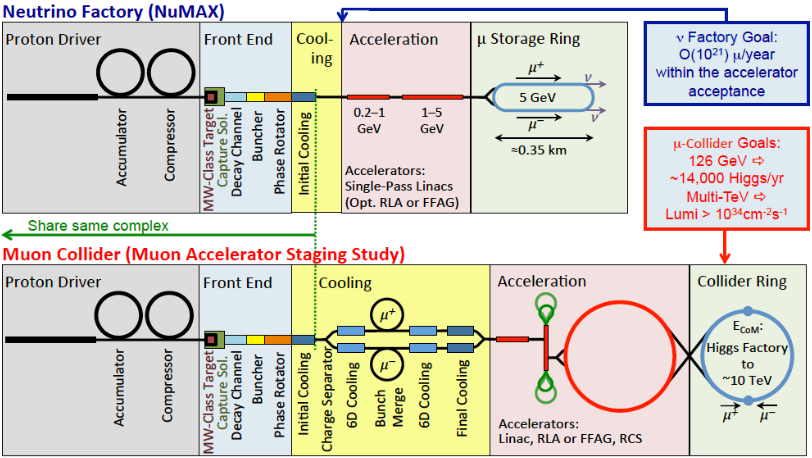
- Neutrino interactions can lead to offsite radiation hazard at very high energies

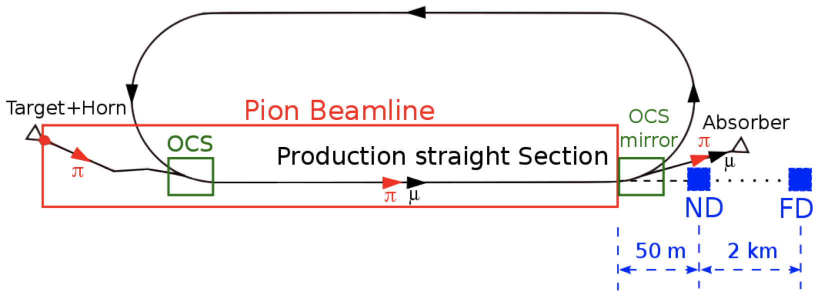
Muon accelerators

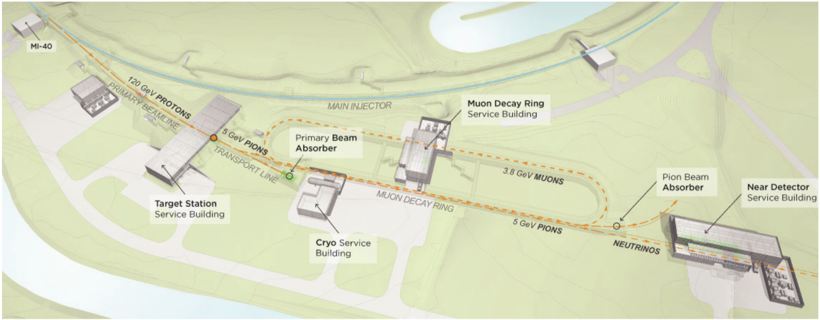
- No natural source for efficient production like e, p
- High-power proton beam on target, solenoidal capture of low-energy charged pions
- which decay into a muon cloud with large phase space volume
- OK for a Neutrino Factory, additional cooling required for a Muon Collider
- Short lifetime means all beam manipulations must be completed before the muons decay
- Ionization cooling the only known practical technique

Muon accelerators

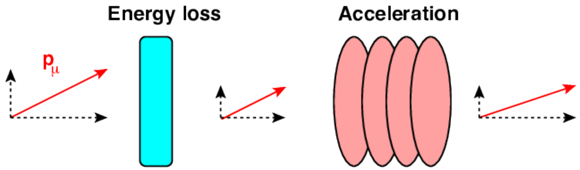


NuSTORM – a hybrid Neutrino Factory





Ionization cooling



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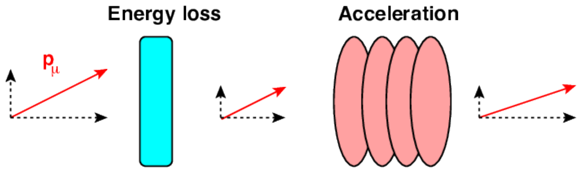
Ionization cooling



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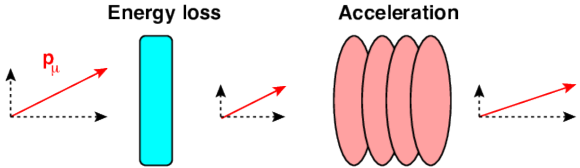

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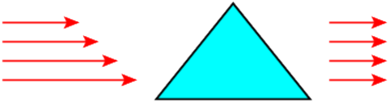
- Longitudinal cooling requires momentum-dependent path-length through absorbers



Ionization Cooling

Normalized transverse emittance ε of muon beam in solenoidal channel

$$\frac{d\varepsilon}{ds} \simeq -\frac{\left\langle \frac{dE}{ds} \right\rangle}{\beta^2 E}(\varepsilon - \varepsilon_0), \quad \varepsilon_0 \simeq \frac{0.875\text{MeV}}{\left\langle \frac{dE}{ds} \right\rangle X_0} \frac{\beta_\perp}{\beta}$$

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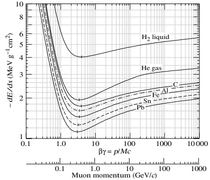
- Low muon momentum



References

https://iopscience.iop.org/journal/1748-0221/page/extraproc46

Journal of Instrumentation

Muon Accelerators for Particle Physics (MUON)

Muon accelerators offer unique potential for particle physics applications. The decay of muon beams within a storage ring can provide pure, well-characterized and intense neutrino beams for short- and long-baseline neutrino-oscillation studies—thus providing measurements of key parameters, such as the CP-violating phase, with unmatched precision and uniquely sensitive probes for new physics. Muon beams are not subject to the synchrotron radiation and beamstrahlung limits imposed on electron-positron colliders because the muon mass is 200 times that of the electron. Thus muon beams can be accelerated to TeV-scale energies and stored in collider rings where the beams can interact for many revolutions. For center-of-mass energies >1 TeV, muon colliders provide the most power efficient route to providing a high luminosity lepton collider.

The concept of the muon collider (MC) was first proposed in 1969[1], while the concept for the neutrino factory (NF) appeared in 1997[2]. The original design concepts have been developed through a series of design studies and a program of accelerator R&D has been carried out to lay the groundwork for deploying these next-generation particle physics capabilities. This volume summarizes work that has been carried out by the U.S. Muon Accelerator Program (MAP)[3], the International Design Study for a Neutrino Factory (IDS-NF)[4], and the international Muon Ionization Cooling Experiment (MICE)[5] to establish the design concepts and to carry out the required feasibility R&D for these machines. It summarizes the current state of the designs for short- and long-baseline neutrino factories (including the nuSTORM short-baseline NF, the IDS-NF reference design and the NuMAX long-baseline concept) as well as the current collider concepts. It also summarizes the status of the technology R&D that has been carried out to allow these capabilities to be deployed and, in particular, the efforts underway at MICE to demonstrate the feasibility of producing cooled muon beams.

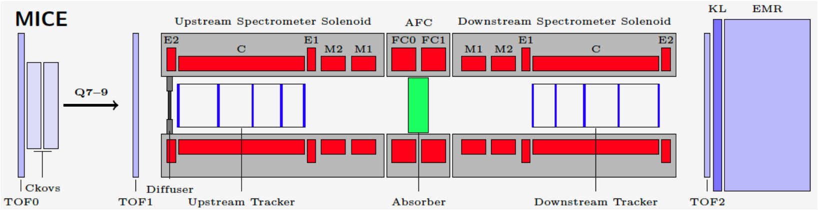
Dr. Palmer[a] and Prof. Long[b]

Muon Ionization Cooling Experiment

- Demonstration of ionization cooling in a setting relevant to muon accelerators
  - measure performance in various modes of operation and beam conditions, thereby investigating the limits and practicality of cooling
  - study aspects critical to performance (multiple scattering, energy loss, phase space evolution)
  - validate design & simulation tools
- Concept
  - track each muon before & after cooling hardware
  - form virtual beams in offline software
  - designed for measuring relative change in emittance to 1%
  - accelerator R&D in the form of a particle physics experiment

Layout

Particle ID – Momentum measurement – Cooling – Momentum measurement – Particle ID



- LiH, LH2, polyethylene (wedge) absorbers
- solenoid (same sign coils) and (sign) flip optics modes

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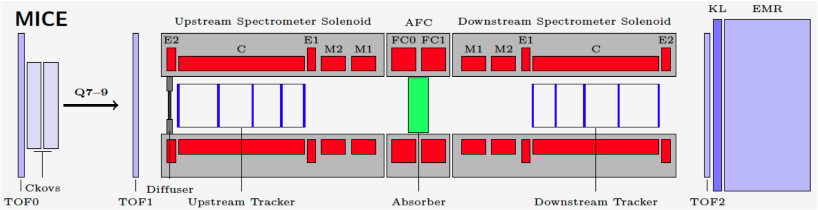

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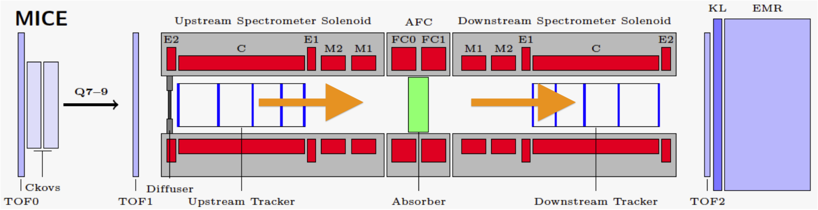

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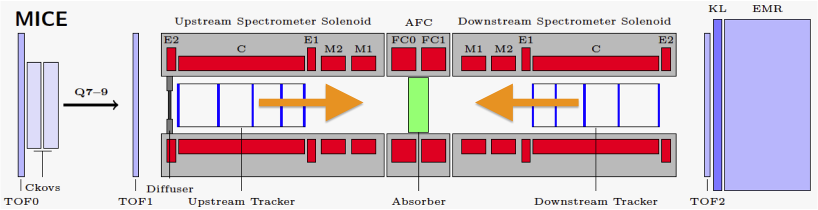

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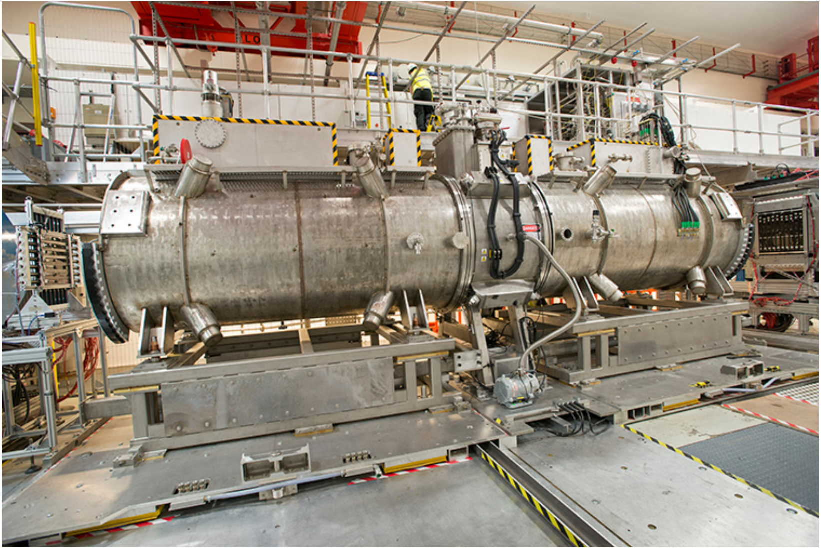
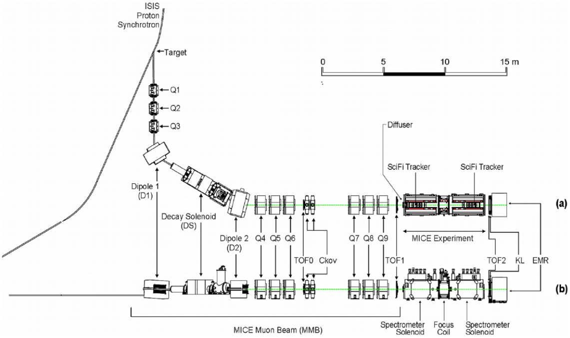


Beamline



- Ti target dips into ISIS halo to produce pions
- muon decay solenoid, momentum selection, focusing
- 120-260 MeV/c muons, >99% purity
- 2-10mm emittance using diffuser

Cooling apparatus

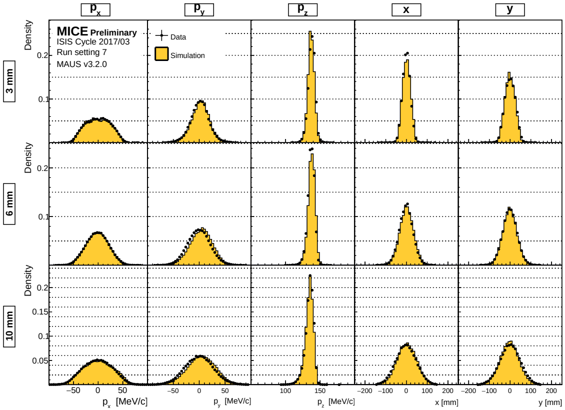
- Absorbers
  - 35-cm-thick LH2
  - 6.5-cm-thick LiH
  - polyethylene 45^o-wedge
- contained within dual-coil magnet for low-$\beta_\perp$ focus
- 40-cm-bore 5-coil spectrometer solenoids providing uniform 4T-field for momentum measurement

Detectors

- 2 threshold Cherenkov counters upstream
- 3 ToF walls with 2 planes of scintillator bars each
- 2 scintillating fiber trackers to reconstruct helical path, each with 5 stations and 3 doublet views/station using 0.35mm fibers and VLPC readout (D0 technology)
- Lead-scintillator preshower detector (KL) downstream
- Totally active scintillator calorimeter (EMR) downstream

Analysis status

- Full suite of tools with detailed material and field distributions
- Excellent agreement for beam profiles (upstream/downstream), transmission



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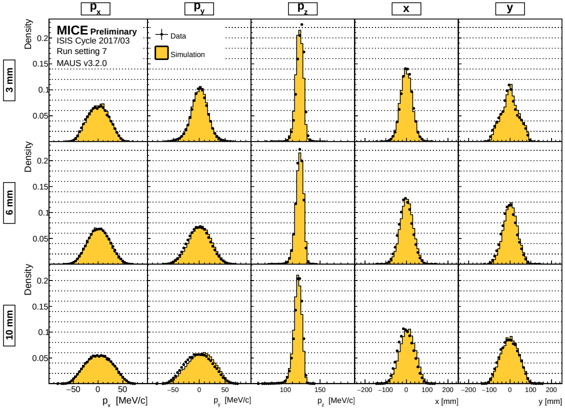

Measuring beam cooling

- Transverse normalized emittance commonly used to characterize phase space volume
  - works well for Gaussian beam through linear optics with no losses
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- Use quantities that are robust and relevant
  - transverse amplitude
  - subemittance, fractional emittance
  - phase space density, core volume

Emittance

Normalized RMS transverse 4D emittance $\epsilon_\perp$

$$\epsilon_\perp = \frac{1}{m_\mu c}|\Sigma|^{1/4}$$

defined through phase space covariance matrix Σ

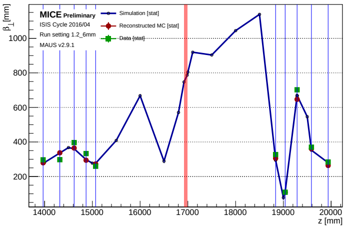
$$\Sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xp_x} & \sigma_{xy} & \sigma_{xp_y} \\ \sigma_{p_x x} & \sigma_{p_x p_x} & \sigma_{p_x y} & \sigma_{p_x p_y} \\ \sigma_{yx} & \sigma_{yp_x} & \sigma_{yy} & \sigma_{yp_y} \\ \sigma_{p_y x} & \sigma_{p_y p_x} & \sigma_{p_y y} & \sigma_{p_y p_y} \end{pmatrix}, \quad \sigma_{ab} = \langle (a - \langle a \rangle)(b - \langle b \rangle) \rangle$$

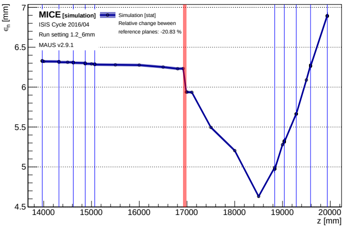
corresponds to volume V of 4D rms ellipsoid and indicates an average phase space density

$$\rho = \frac{N}{V} = \frac{2}{\pi^2} \frac{N}{|\Sigma|^{1/2}}$$

Evolution of RMS emittance

- solenoid mode optics, LiH absorber, 6-mm 140-MeV/c input beam
- limited transmission + betatron motion
  $\Rightarrow$ large apparent cooling at downstream tracker plane
- rms emittance is a poor indicator in this case





Transverse single-particle amplitude

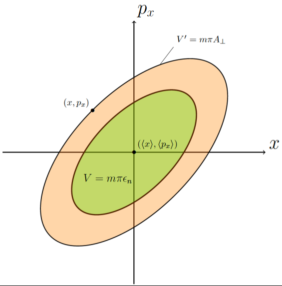
Defined as

$$A_\perp = \epsilon_\perp \, \mathbf{u}^T \Sigma^{-1} \, \mathbf{u}$$

for centered phase space coordinates

$$\mathbf{v} = (x, p_x, y, p_y)$$

$$\mathbf{u} = \mathbf{v} - \langle \mathbf{v} \rangle$$



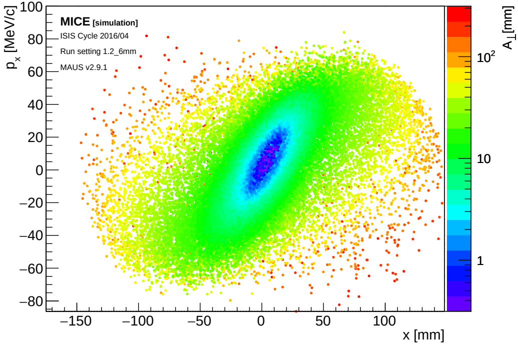
- Associated with phase space volume similar to rms ellipsoid (emittance)
- Provides density estimate at every sample point

$$\rho(\mathbf{v}_i) = \rho_0 \, exp\left[-\frac{1}{2}\frac{A_\perp}{\epsilon_\perp}\right]$$

- Allows identification of
  low $A_\perp \Leftrightarrow$ high ρ core
  high $A_\perp \Leftrightarrow$ low ρ tail
- Highest amplitude particles can be removed iteratively to prevent bias

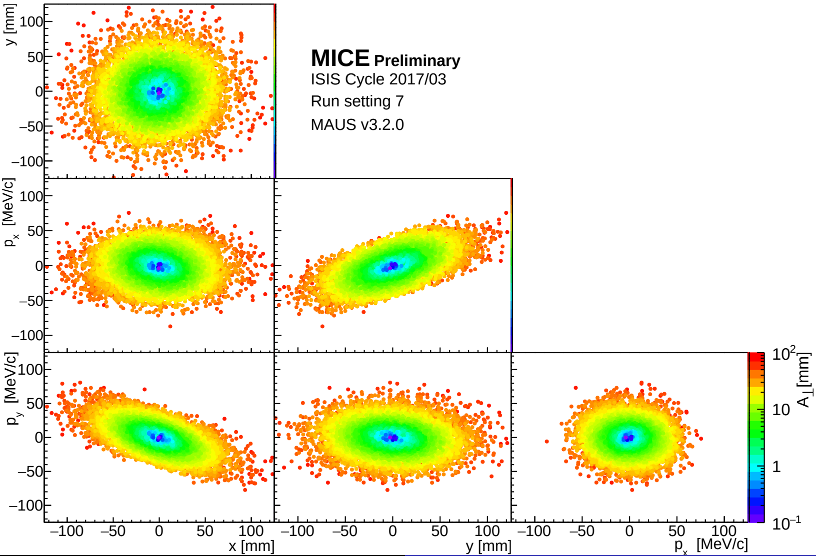
Amplitude reconstruction example [simulation]

- 6-mm 140-MeV/c input beam, solenoid mode optics
- Last (most downstream) measurement plane

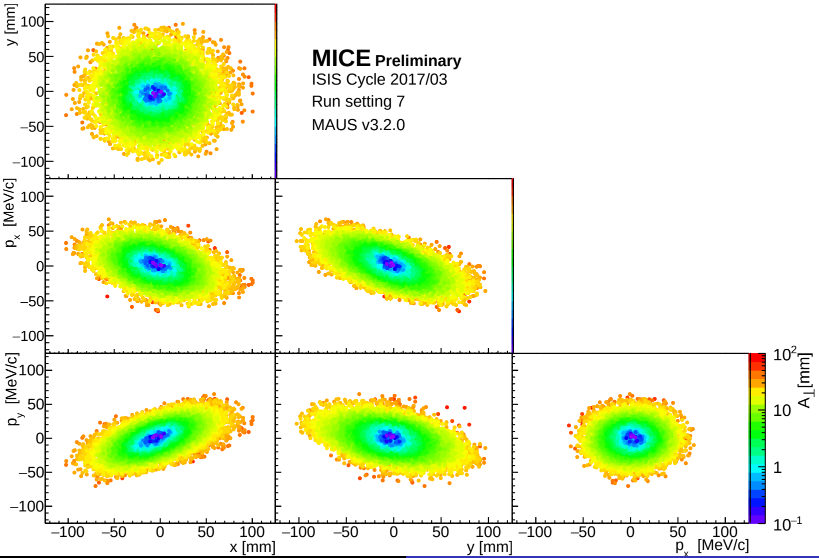


- High-density low-amplitude (cool) Gaussian core
- Low-density high-amplitude (hot) tails

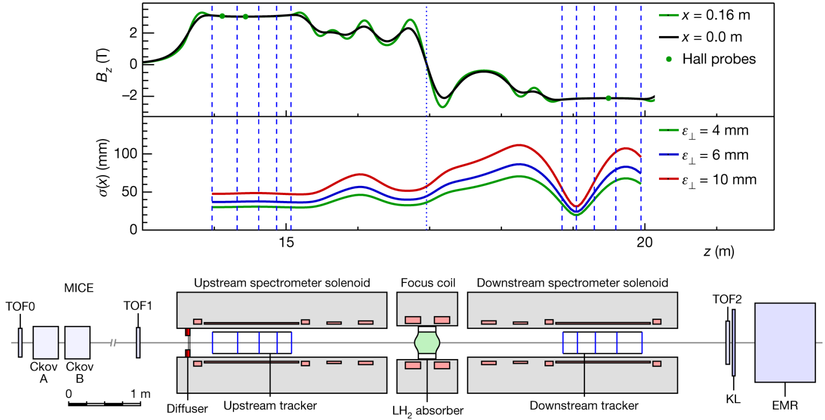
Poincaré sections [data] (upstream)
6-mm 140-MeV/c beam – flip mode – LiH



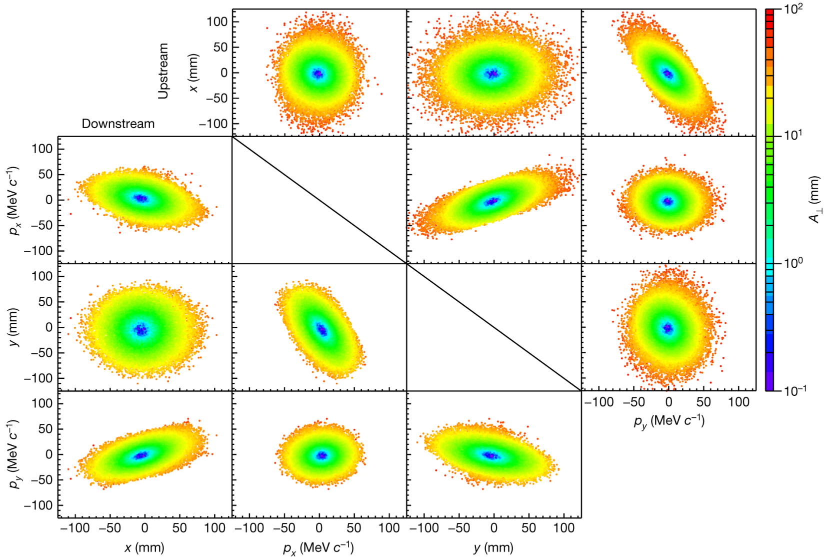
Poincaré sections [data] (downstream)
6-mm 140-MeV/c beam – flip mode – LiH



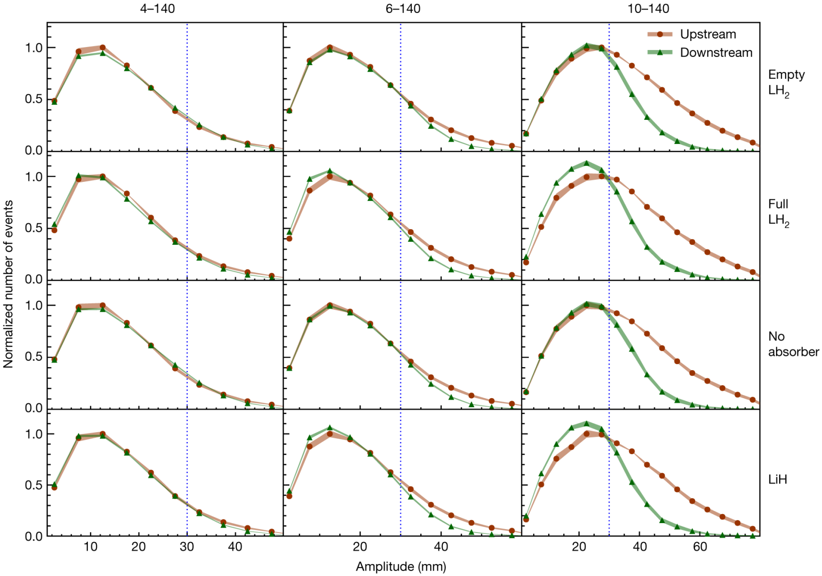
Layout and Magnetic Field



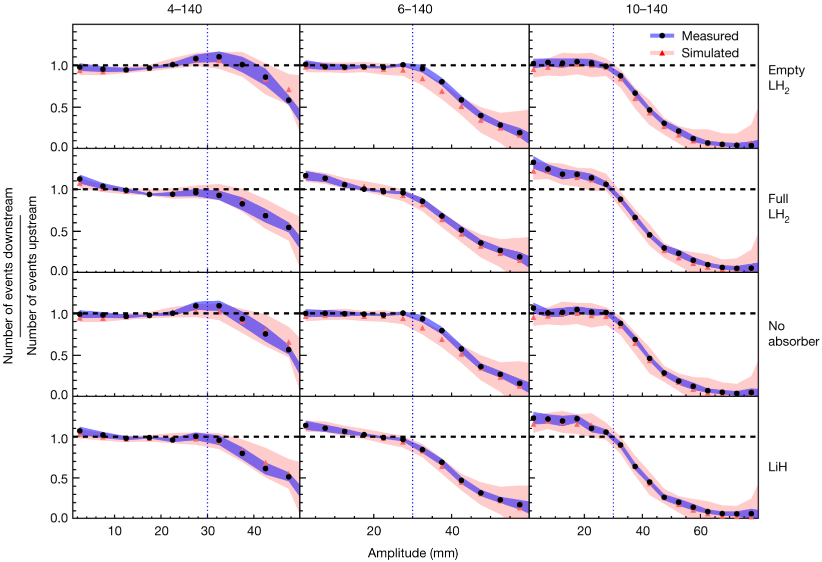
Phase Space – 6-140 Setting – LH2



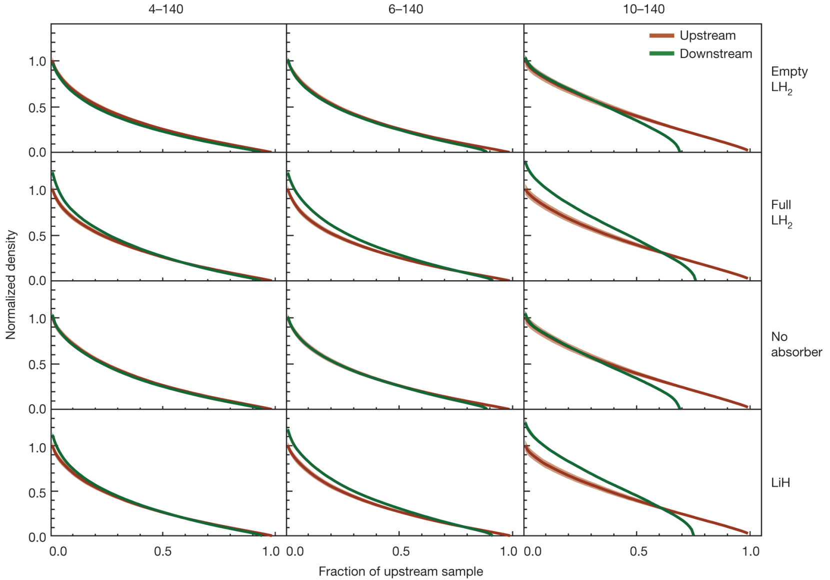
Amplitude Distributions



Amplitude Distribution Ratios



Density Quantiles



Outlook

- Unique single-particle measurement capabilities, large data sets and mature analysis tools of MICE allow detailed studies of the beam phase space
    - Amplitude based analysis used to avoid artifacts due to nonlinear transport
    - Core density/volume used for selecting the portion of the beam that is transmitted
    - Non-parametric density estimators substantially independent of the underlying distribution

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  - Non-parametric density estimators substantially independent of the underlying distribution
- Successful demonstration of ionization cooling in realistic environment
  - Techniques/results directly applicable to practical muon accelerators

RF in MICE

- Initial MICE design included 201-MHz RF acceleration modules
- An RF module prototype was assembled and tested at Fermilab up to 50% higher than design gradient
- 2 production modules were built at LBNL and crated up for shipping
- Experiment was de-scoped and the RF stage removed from the timeline

Subemittance

- For a parent beam of n particles, select a fraction α from the core
- α-amplitude A_α is the largest amplitude in the α-sample

$$A_\alpha = \epsilon_\perp \text{ at } \alpha\text{=9\% for Gaussian beam in 4D}$$

- 9% is the 1-σ volume fraction in 4D
- α-subemittance e_α is defined as the rms emittance of the α-sample

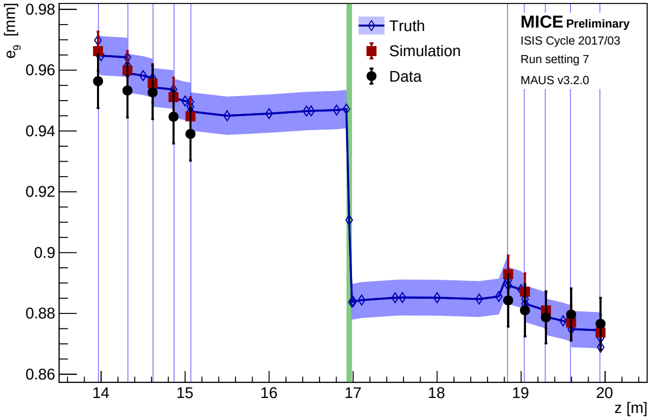
$$e_\alpha \leq \epsilon_\perp$$

- If an identical fraction α is selected upstream and downstream

$$\frac{\Delta A_\alpha}{A_\alpha} = \frac{\Delta e_\alpha}{e_\alpha} = \frac{\Delta \epsilon_\perp}{\epsilon_\perp}$$

for Gaussian core with full transmission

Subemittance evolution
6-mm 140-MeV/c beam – flip mode – LiH

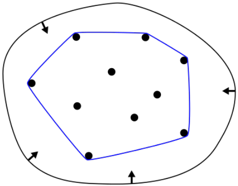


Fractional emittance

- The α-fractional emittance ϵ_α is defined as the phase space volume occupied by the core fraction α of the parent beam.
- Found by calculating the volume of the convex hull of the α-sample (smallest convex set containing all the points)

- For $\alpha = 9\%$

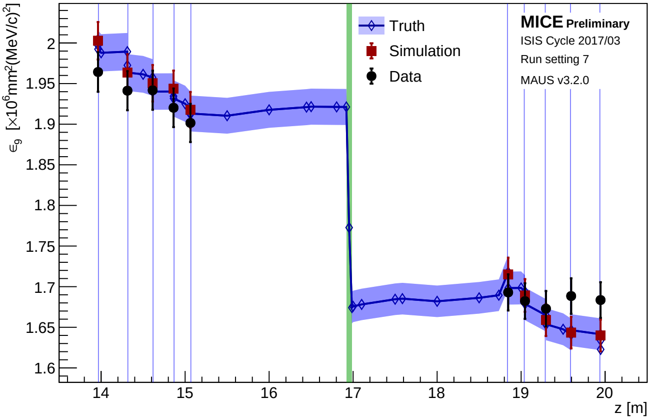
$$\epsilon_\alpha = \frac{1}{2}(\pi\, m\, c\, \epsilon_\perp)^2$$



- For small change

$$\delta = \frac{\Delta\epsilon_\perp}{\epsilon_\perp} \ll 1 \rightarrow \frac{\Delta\epsilon_\alpha}{\epsilon_\alpha} \simeq 2\delta$$

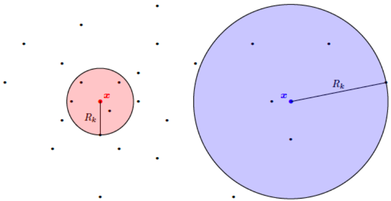
Fractional (9%) emittance evolution
6-mm 140-MeV/c beam – flip mode – LiH

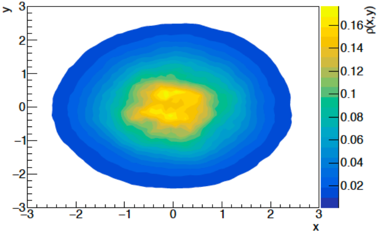


Non-parametric density estimation

- Amplitude based methods work well for Gaussian core, small fraction of a nonlinear beam
- Non-parametric density estimators can be used to extend the analysis
- Several methods considered including
  - optimally binned histograms
  - k-nearest neighbors (kNN)
  - tessellation density estimators (TDEs)
  - kernel density estimation (KDE)
- kNN and KDE examples follow

k-Nearest neighbor algorithm





kNN density of 10^3 Gaussian points

To find the density $\rho(\mathbf{x})$ at a point $\mathbf{x}$ in phase space, identify nearby data points $\mathbf{x}_i$. Using the distance R_k to the kth-nearest point

$$\rho(\mathbf{x}) = \frac{k}{V(R_k)}$$

where $V(R_k)$ is the volume of the 4-ball with radius R_k

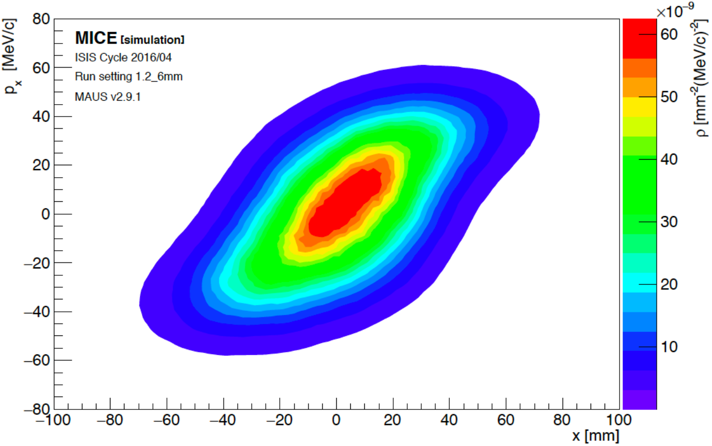
$$V = \pi^2 R_k^4/2$$

Near optimal results for

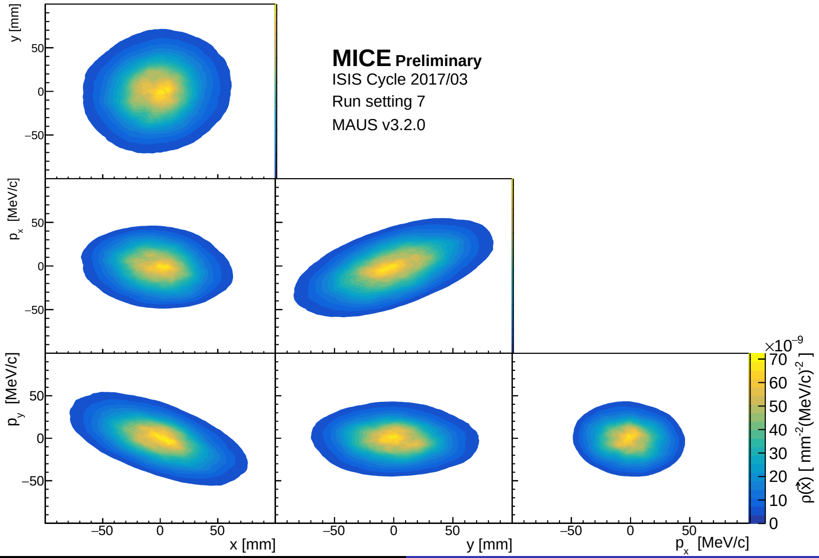
$$k = \sqrt{n}$$

kNN density estimate [simulation]

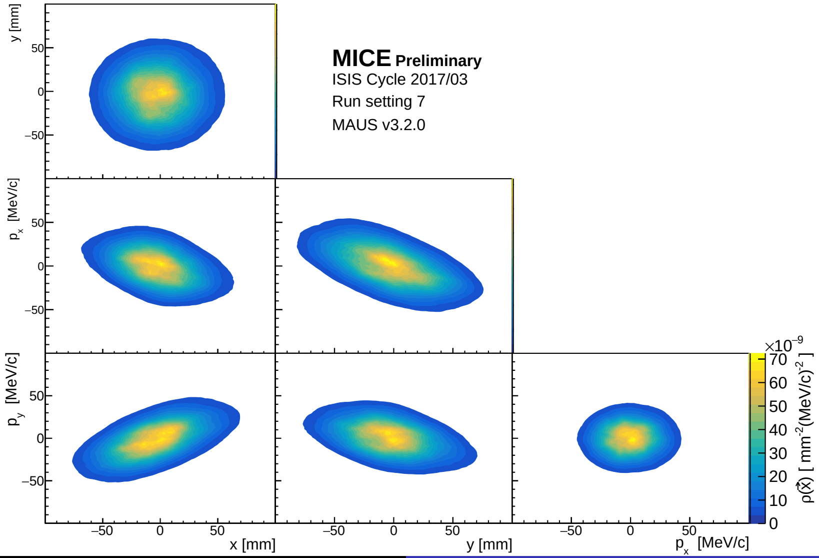
- 6-mm 140-MeV/c input beam, solenoid mode optics
- Last (most downstream) tracker plane
- reconstructed 4D density projected to $(y, p_y) = (0, 0)$



Poincaré sections [kNN + data] (upstream)
6-mm 140-MeV/c beam – solenoid mode – LiH



Poincaré sections [kNN + data] (downstream)
6-mm 140-MeV/c beam – solenoid mode – LiH

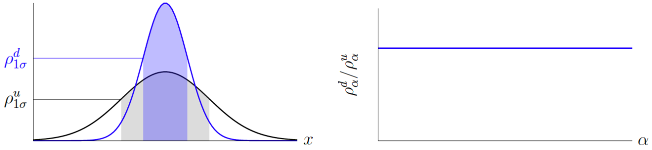


Contour levels

- Given the cumulative distribution function F for the beam
- find the density level ρ_α (α-quantile, inverse of CDF) for the contour that encloses core fraction α of the beam

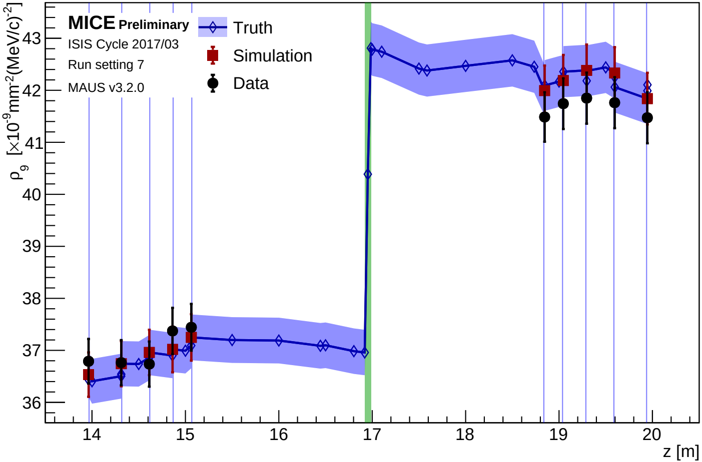
$$\rho_\alpha = \rho(F^{-1}(\alpha))$$

- The evolution of ρ_α shows cooling (ratio independent of α in any dimension for purely Gaussian input/output beams)

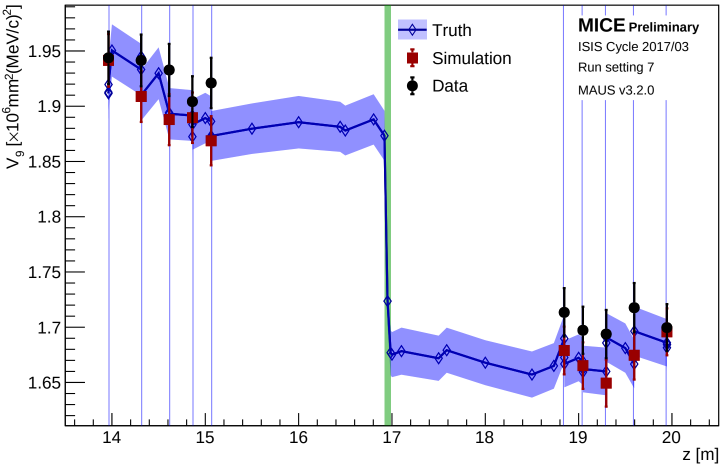


- Can also use the volume of phase space V_α that has $\rho > \rho_\alpha$

(9%) Contour density evolution (kNN)
6-mm 140-MeV/c beam – LiH – flip



(9%) Contour volume evolution (kNN)
6-mm 140-MeV/c beam – LiH – flip



Density estimate ρ at point x

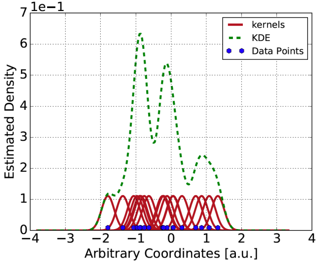
$$\rho(x) = \frac{1}{nh}\sum_{i=1}^{n} K(\frac{x - x_i}{h})$$

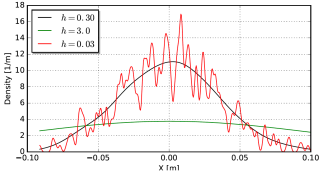
where K is called the kernel function

$$\int K(x)\, dx = 1$$

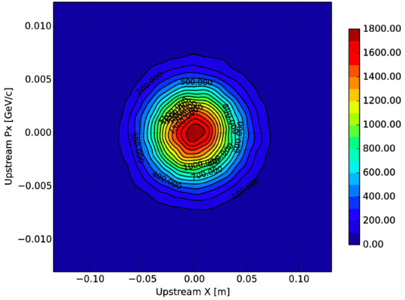
and h, the bandwidth parameter. For d-dimensional phase space, use Gaussian kernel

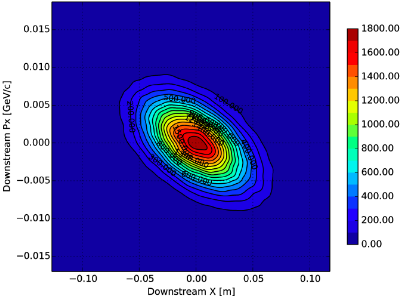
$$\rho(\mathbf{x}) \propto \sum_i exp\left[-\frac{1}{2}(\mathbf{x} - \mathbf{x}_i)^T \Sigma^{-1} (\mathbf{x} - \mathbf{x}_i)\right]$$

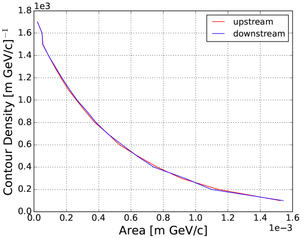




KDE example

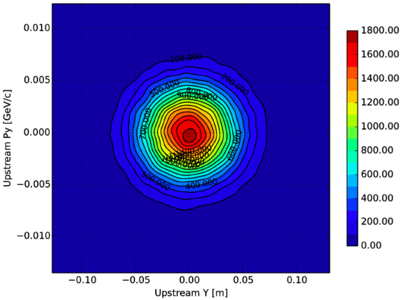


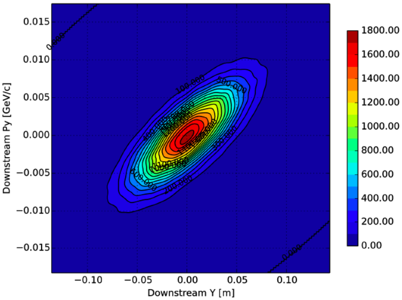


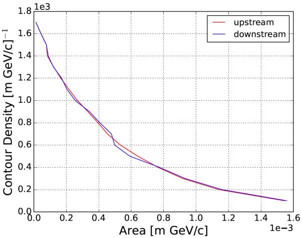


- Gaussian beam with 100k muons through quadrupole
- evaluated on 1k x 1k grid
- 2D contour density and area conserved

KDE example



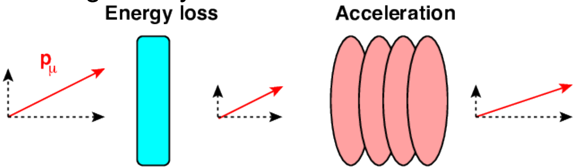




- Gaussian beam with 100k muons through quadrupole
- evaluated on 1k x 1k grid
- 2D contour density and area conserved

Emittance exchange

- Cooling mainly transverse in a linear channel



Emittance exchange

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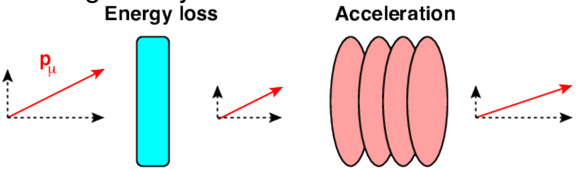
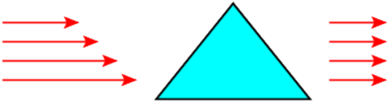


- Longitudinal cooling requires momentum-dependent path-length through the energy absorbers



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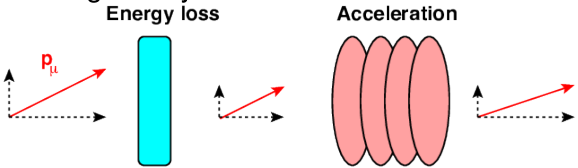

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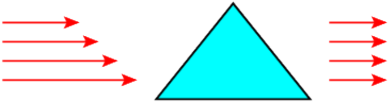
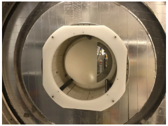
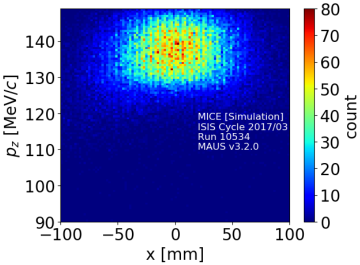
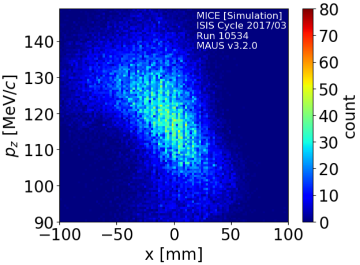


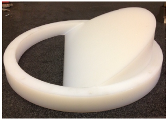


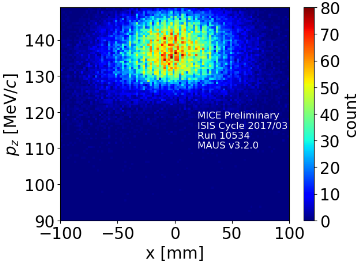
- Wedge shaped polyethylene absorber for demonstration of (reverse) emittance exchange in MICE

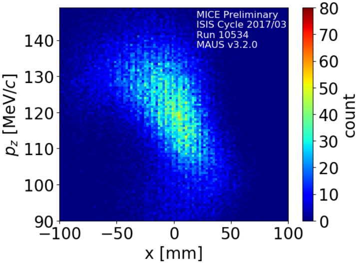
Reverse emittance exchange
6mm 140-MeV/c beam – polyethylene wedge



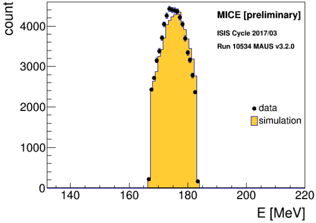


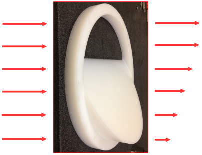


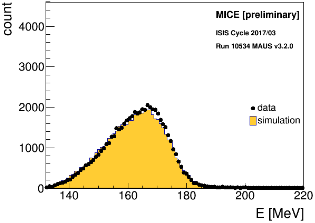




Reverse emittance exchange
6mm 140-MeV/c beam – polyethylene wedge

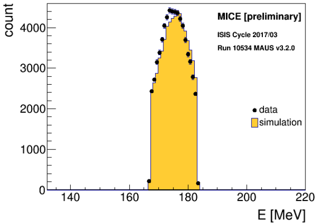


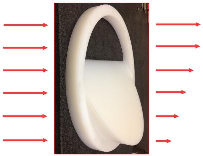


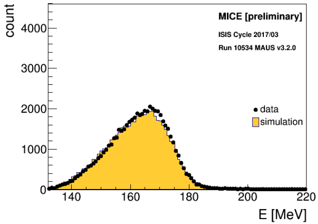


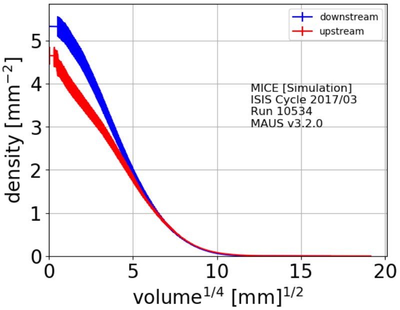
- No RF $\rightarrow$ longitudinal space is 1D (E)
  - Longitudinal heating

Reverse emittance exchange
6mm 140-MeV/c beam – polyethylene wedge









- No RF → longitudinal space is 1D (E)
  - Longitudinal heating
- 4D transverse phase space $(x, p_x/\langle p\rangle, y, p_y/\langle p\rangle)$
  - Transverse cooling