Proposal for a Sub-Set of 20 Vertical Trim Quads for Booster Tune Adjustment

John A. Johnstone
04.26.2020

In the PIP-II era Booster injection will move to Long Straight 11. Without significant lattice modifications, though, the greatly increased beam power will create radiation problems. Even replacing the magnets at the end of the straight with shorter designs, the dearth of space remaining to accommodate an adequate absorber still results in 1.6 R/hr on the downstream corrector with the latest absorber design[1]. Interest has been expressed, therefore, in eliminating the corrector currently at L11, replacing it with a much shorter horizontal & vertical dipole corrector package.

It is the purpose of the present note to explore whether the quadrupole function of the L11 corrector for tune adjustment can be bypassed.

†

Consider quadrupole contributions distributed around the base ring lattice, where these quadrupole terms are defined in thin-lens approximation by[2]:

$$\bar{q}_i \equiv \begin{pmatrix} 1 & 0 \\ q_i & 1 \end{pmatrix}$$

Then with $\bar{x}$, $\bar{x}'$ the generalized phase-space co-ordinates: $\bar{x} = x / \sqrt{\beta}$, and $\bar{x}' = (\beta x' + \alpha x) / \sqrt{\beta}$, and the unperturbed ring transfer matrix:

$$\bar{M}_0 \equiv \begin{pmatrix} \cos\upsilon_0 & \sin\upsilon_0 \\ -\sin\upsilon_0 & \cos\upsilon_0 \end{pmatrix}$$

It can be shown (details will be presented elsewhere[3]) that the transport of a particle once around the ring can be described exactly in terms of the *unperturbed* lattice functions by:

$$\boxed{\begin{pmatrix} \bar{x}_f \\ \bar{x}'_f \end{pmatrix} = \bar{M}_0 \cdot \prod_{i=1}^{N} \left(1 + {}^{q_i\beta_i}\!/_2 \cdot \begin{pmatrix} -\sin 2\psi_i & -1+\cos 2\psi_i \\ 1+\cos 2\psi_i & \sin 2\psi_i \end{pmatrix} \right) \cdot \begin{pmatrix} \bar{x}_0 \\ \bar{x}'_0 \end{pmatrix}}$$

---

[1] Dave Johnson, private communication.
[2] The extension to distributed, rather than point-like, quadrupole sources presents purely a technical complication and does not introduce additional physics issues.
[3] elsewhere : not here.

With the assumption that the quadrupole errors are 'small', the product above can be approximated by terms linear in the q_i by:

$$\prod_{i=1}^{N}\left(1+ {}^{q_i\beta_i}\!/_{2} \cdot \begin{pmatrix} -\sin 2\psi_i & -1+\cos 2\psi_i \\ 1+\cos 2\psi_i & \sin 2\psi_i \end{pmatrix}\right) \approx \mathbf{1} + \sum_{i=1}^{N} {}^{q_i\beta_i}\!/_{2} \cdot \begin{pmatrix} -\sin 2\psi_i & -1+\cos 2\psi_i \\ 1+\cos 2\psi_i & \sin 2\psi_i \end{pmatrix}$$

Consider the case in which there are N quad terms, all of which are equal strength, located at equal values of β. The linear terms from above then become:

$$1 + N\frac{q\beta}{2} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \frac{q\beta}{2}\sum_{i=1}^{N}\begin{pmatrix} -\sin 2\psi_i & \cos 2\psi_i \\ \cos 2\psi_i & \sin 2\psi_i \end{pmatrix}$$

The first 2 terms lead to a linear tune shift:

$$\begin{pmatrix} \cos\nu & \sin\nu \\ -\sin\nu & \cos\nu \end{pmatrix} \approx \bar{M}_0 \cdot \left[1 + N\frac{q\beta}{2} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\right]$$

$$= \begin{pmatrix} \cos\nu_0 + N\frac{q\beta}{2} \cdot \sin\nu_0 & \sin\nu_0 - N\frac{q\beta}{2} \cdot \cos\nu_0 \\ -\sin\nu_0 + N\frac{q\beta}{2} \cdot \cos\nu_0 & \cos\nu_0 + N\frac{q\beta}{2} \cdot \sin\nu_0 \end{pmatrix}$$

From which, using the small angle approximation $\sin\theta \approx \theta$, the tune shift is given by:

$$\boxed{\Delta\mu \approx -N\frac{q\beta}{4\pi}}$$

The third linear term, which depends on the betatron phase advance ψ_i to the qi's, contributes to the perturbed ring transfer matrix as:

$$\bar{M}_0 \cdot \frac{q\beta}{2}\sum_{i=1}^{N}\begin{pmatrix} -\sin 2\psi_i & \cos 2\psi_i \\ \cos 2\psi_i & \sin 2\psi_i \end{pmatrix}$$

$$= \frac{q\beta}{2}\sum_{i=1}^{N}\begin{pmatrix} \sin(\nu_0 - 2\psi_i) & \cos(\nu_0 - 2\psi_i) \\ \cos(\nu_0 - 2\psi_i) & -\sin(\nu_0 - 2\psi_i) \end{pmatrix}$$

It is left as an exercise for the interested reader[4] to show that this term does not impact the tune. It does, however, introduce a beta-wave. Accumulating the bits & pieces of the linear quad terms, the perturbed ring-wide transfer matrix has the form:

$$\bar{M} \approx \begin{pmatrix} \cos\nu + \frac{q\beta}{2}\sum_{i=1}^{N}\sin(\nu_0 - 2\psi_i) & \sin\nu + \frac{q\beta}{2}\sum_{i=1}^{N}\cos(\nu_0 - 2\psi_i) \\ -\sin\nu + \frac{q\beta}{2}\sum_{i=1}^{N}\cos(\nu_0 - 2\psi_i) & \cos\nu - \frac{q\beta}{2}\sum_{i=1}^{N}\sin(\nu_0 - 2\psi_i) \end{pmatrix}$$

From which it is concluded that a β-wave is produced, the magnitude of which is dependent on the betatron phases of the perturbing quads, i.e;

$$\boxed{\frac{\Delta\beta}{\beta} \approx \frac{q\beta}{2\sin\nu}\cdot\sum_{i=1}^{N}\cos(\nu_0 - 2\psi_i)}$$

The simplest approach for eliminating this linear β-wave contribution is to choose sets of quadrupoles such that[5]:

$$\sum_{i=1}^{N}\cos(2\psi_i) \equiv 0\,, \quad \text{and} \quad \sum_{i=1}^{N}\sin(2\psi_i) \equiv 0$$

In the Booster design lattice each cell has a phase advance of 100.5° for a machine tune of 6.700. This is close enough to the 2/3 resonance that it's reasonable to assume the cells are well approximated by phase advance ≈ 100.0^0. With this simplification there are a variety of quadrupole sets that satisfy the requirements that the sine & cosine summations are separately equal to zero. The configuration considered here is based on a set of 10 quads, spaced by 1 cell. One such set are the quads QL01 →QL10. This scheme is extended to include the set QL13 →QL22, thereby doubling the number of quads to 20 to more evenly distribute their effect.

The 20 quad sub-set for tune adjustment is, therefore, obtained by throwing away all the quad contributions from the 4 trims QL11, QL12, QL23, & QL24.

†

A 1st look at vertical tune adjustment using all 24 trim qv's *vs* the proposed scheme with 20 qv's is considered below. In MAD-X, with the 4 quads mentioned zeroed out, the remaining quads had the horizontal & vertical k1 values augmented by khtune & kvtune. As such, khtune & kvtune are in units of

[4] Assuming, optimistically, that such an entity exists.

[5] Note, however, this does not preclude generation of a dispersion wave: η waves propagate linearly with phase advance, unlike β errors that advance as twice the phase.

m^-2. The machine was then re-tuned from nominal values to (rather arbitrarily) Q1=6.7598, Q2=6.6000.

A comparison of relevant lattice parameters between the nominal tune values, and those after vertical re-tuning with the 24 & 20 vertical quad sets appear in Table 1 for a sampling of energies across the acceleration cycle. This is not intended to be exhaustive – particularly as simulations were only performed for the one set of tunes mentioned – it only represents a casual cruise through parameter space.

Staring at the table entries reveals that at lower energies the 20 quad sub-set solution is slightly better than both the nominal lattice & the 24 quad results as measured by maxima in βx, βy, and ηx. As energy increases somewhere beyond the neighborhood of $\approx 1 \rightarrow 2$ GeV the 24 quad results become very marginally better than the 20 quad set. The origin of this curious trend is not currently understood, but in practical terms these small differences are effectively irrelevant. The difference in RMS η values is measured in millimeters and, since the beam is shrinking adiabatically, small variations in β become increasingly unimportant with acceleration. For example, in timeslot = 31 a 16π emittance vertical beam has a maximum σy = 2.6 mm and the difference between the 24 & 20 quad solutions is only 16 µm.

Lattice functions for the 24 & 20 quad solutions for vertical tune adjustment at the extremes of the range considered – timeSlots 3 & 31 -- are compared in Figures 1 & 2.

†

This exercise was repeated to adjust the horizontal tune while keeping the vertical constant. In this case the machine was re-tuned to drop the horizontal tune close to the half-integer resonance, with test case values of Q1=6.5500, Q2=6.4875.

A comparison of relevant lattice parameters between the nominal tune values, and those after horizontal re-tuning with the 24 & 20 vertical quad sets appear in Table 2. Essentially the same observations about these results apply as those discussed for vertical tune adjustment, i.e; the very small differences between βx, βy, and ηx values in the 24 & 20 quad sets are inconsequential for all practical purposes.

Lattice functions for the 24 & 20 quad solutions for horizontal tune adjustment at the extremes of the range considered – timeSlots 3 & 31 -- are compared in Figures 3 & 4.

†

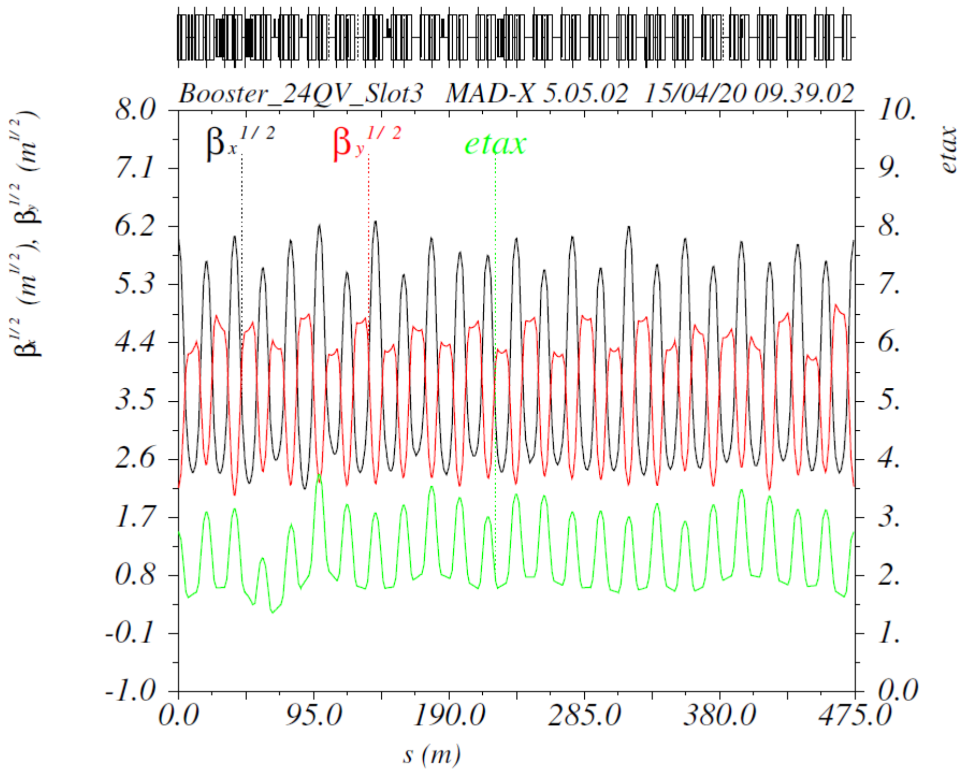
Based on the results of the current study it is concluded that the vertical quad function at QL11 for tune adjustment can be eliminated with impunity by removing all vertical quad contributions from Q11, Q12, Q23, & Q24.

Ω

VERTICAL TUNE ADJUSTMENT WITH 20 VS 24 VERTICAL QUAD SETS

timeSlot	KE (GeV)		β_X(max) (m)	β_Y(max) (m)	η_X(max) (m)	η_X(rms) (m)	k_H (m^{-2})	k_V (m^{-2})
3	0.41527	Nominal	39.5107	24.9529	3.7386	2.4416	0	0
		24 QV	39.7615	27.4937	3.7607	2.4169	-0.06081	0.27949
		20 QV	39.4081	27.2967	3.5577	2.4139	-0.06120	0.33496
7	0.76183	Nominal	37.5661	23.1101	3.4355	2.4168	0	0
		24 QV	37.3932	25.1640	3.4760	2.4161	-0.07631	0.23884
		20 QV	37.1013	25.0635	3.3559	2.4159	-0.07643	0.27745
9	1.08191	Nominal	36.5697	22.5885	3.3440	2.4217	0	0
		24 QV	36.6628	24.0846	3.3856	2.4153	-0.07705	0.26152
		20 QV	36.3480	24.1680	3.3283	2.4158	-0.07716	0.30386
11	1.47975	Nominal	36.0193	22.1568	3.2919	2.4220	0	0
		24 QV	36.1288	23.3747	3.3260	2.4145	-0.07654	0.26408
		20 QV	35.8092	23.5494	3.3132	2.4153	-0.07662	0.30731
15	2.45358	Nominal	35.3117	21.5938	3.2684	2.4274	0	0
		24 QV	35.4776	22.4818	3.2593	2.4137	-0.06697	0.25705
		20 QV	35.1434	22.8826	3.3002	2.4148	-0.06703	0.29838
19	3.57600	Nominal	34.9372	21.3007	3.2476	2.4288	0	0
		24 QV	35.0683	21.9858	3.2264	2.4131	-0.06182	0.24858
		20 QV	34.7790	22.3646	3.2891	2.4142	-0.06187	0.28791
23	5.31907	Nominal	35.2626	21.2666	3.2125	2.3991	0	0
		24 QV	34.7938	21.6472	3.2075	2.4150	-0.10850	0.27768
		20 QV	34.8223	22.0661	3.2836	2.4162	-0.10855	0.31659
27	6.36265	Nominal	34.9131	21.1351	3.2080	2.4102	0	0
		24 QV	34.7243	21.6386	3.2000	2.4136	-0.09113	0.27006
		20 QV	34.9287	21.9693	3.2811	2.4149	-0.09119	0.30982
31	7.20116	Nominal	34.7095	21.0408	3.2119	2.4225	0	0
		24 QV	34.6842	21.6416	3.1965	2.4135	-0.07505	0.26551
		20 QV	35.0030	21.9170	3.2807	2.4148	-0.07511	0.30587

Table 1. Lattice parameters for nominal tune quad settings & after re-tuning to Q1=6.7598, Q2=6.6000 using all 24 vertical trim quads and the 20 quad sub-set that excludes QL11, QL12, QL23, & QL24.



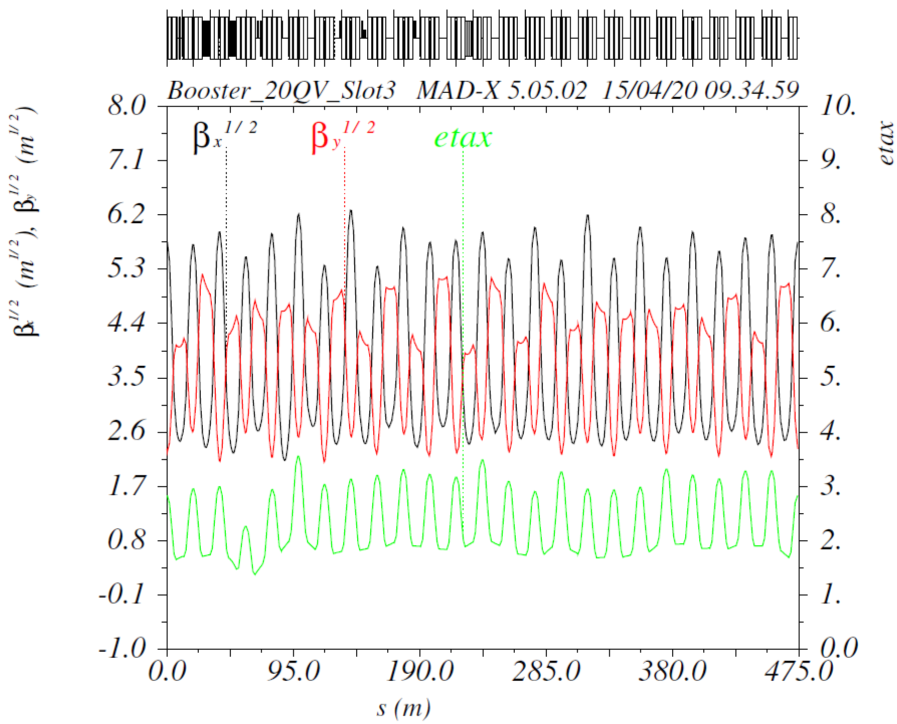


Figure 1. Lattice functions for 24 (top) & 20 (bottom) vertical quad sets at timeslot = 3, and tunes Q1=6.7598, Q2=6.6000.

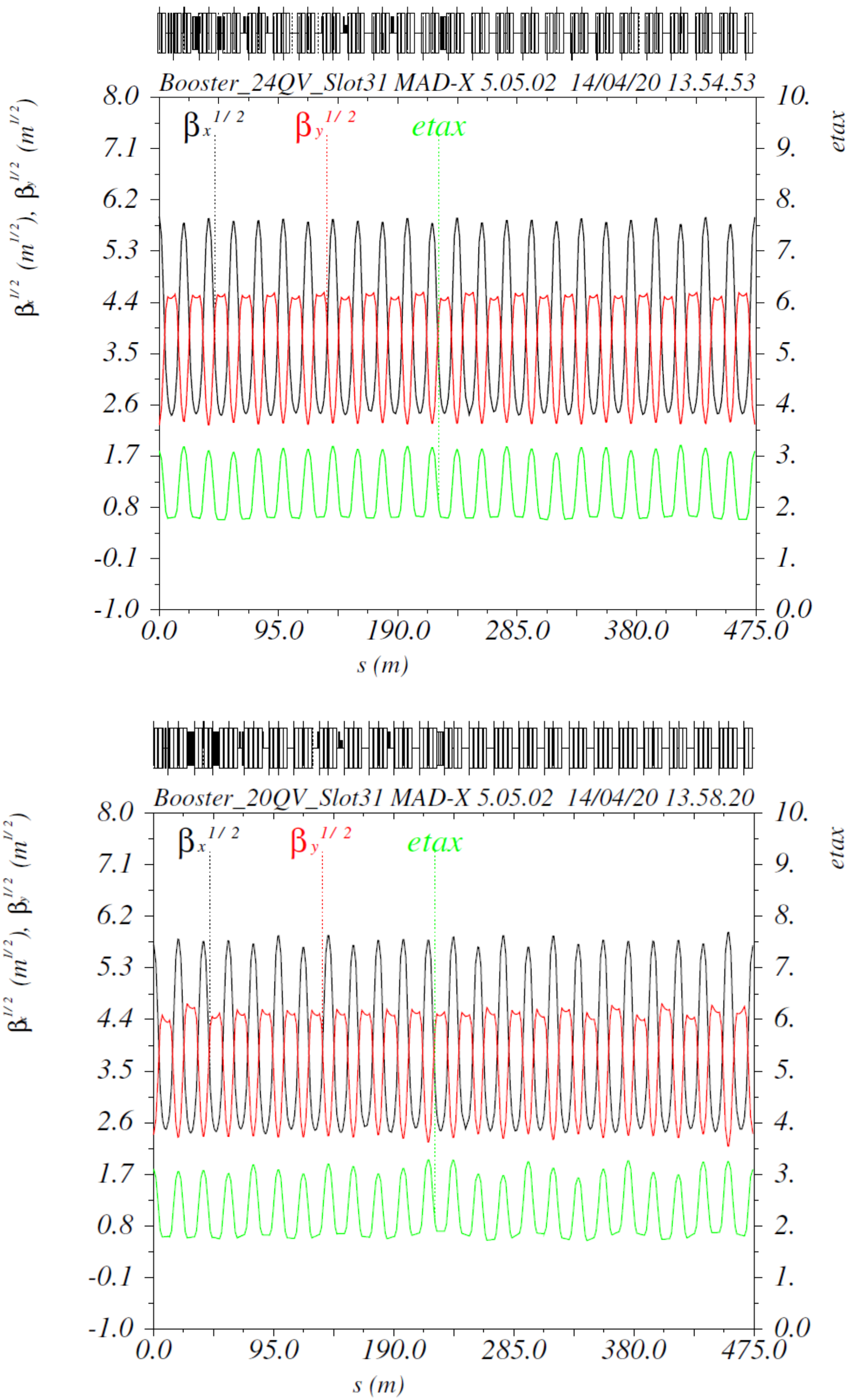


Figure 2. Lattice functions for 24 (top) & 20 (bottom) vertical quad sets at timeslot = 31, and tunes Q1=6.7598, Q2=6.6000.

Horizontal Tune Adjustment with 20 vs 24 Vertical Quad Sets

timeSlot	KE (GeV)		β_X(max) (m)	β_Y(max) (m)	η_X(max) (m)	η_X(rms) (m)	k_H (m^{-2})	k_V (m^{-2})
3	0.41527	Nominal	39.5107	24.9529	3.7386	2.4416	0	0
		24 QV	47.8718	24.8776	3.9106	2.5607	-0.14245	+0.03656
		20 QV	48.2070	23.4931	3.8905	2.5597	-0.14255	+0.04178
7	0.76183	Nominal	37.5661	23.1101	3.4355	2.4168	0	0
		24 QV	41.0409	23.1949	3.6407	2.5573	-0.15689	+0.00397
		20 QV	41.5338	22.8602	3.6868	2.5584	-0.15700	-0.00799
9	1.08191	Nominal	36.5697	22.5885	3.3440	2.4217	0	0
		24 QV	38.8685	22.6111	3.5417	2.5560	-0.15731	+0.02386
		20 QV	39.2079	22.2541	3.5723	2.5566	-0.15738	+0.01802
11	1.47975	Nominal	36.0193	22.1568	3.2919	2.4220	0	0
		24 QV	37.4603	22.1513	3.4885	2.5559	-0.15737	+0.02489
		20 QV	37.7323	21.8876	3.5222	2.5563	-0.15742	+0.01963
15	2.45358	Nominal	35.3117	21.5938	3.2684	2.4274	0	0
		24 QV	35.9662	21.5232	3.4390	2.5568	-0.14906	+0.01528
		20 QV	36.2169	21.5880	3.4789	2.5572	-0.14910	+0.00772
19	3.57600	Nominal	34.9372	21.3007	3.2476	2.4288	0	0
		24 QV	35.3256	21.1606	3.4116	2.5579	-0.14506	+0.24858
		20 QV	34.7790	22.3646	3.4597	2.5582	-0.14510	-0.00529
23	5.31907	Nominal	35.2626	21.2666	3.2125	2.3991	0	0
		24 QV	34.8237	20.8901	3.3842	2.5572	-0.18998	+0.00374
		20 QV	35.0012	21.1875	3.4295	2.5574	-0.19001	+0.02783
27	6.36265	Nominal	34.9131	21.1351	3.2080	2.4102	0	0
		24 QV	34.6293	20.8197	3.3758	2.5582	-0.17417	+0.02628
		20 QV	34.7838	21.1088	3.4200	2.5584	-0.17420	+0.01680
31	7.20116	Nominal	34.7095	21.0408	3.2119	2.4225	0	0
		24 QV	34.5054	20.7731	3.3697	2.5585	-0.15833	+0.02106
		20 QV	34.6288	21.0438	3.4116	2.5587	-0.15836	+0.01204

Table 2. Lattice parameters for nominal tune quad settings & after re-tuning to Q1=6.5500, Q2=6.8475 using all 24 vertical trim quads and the 20 quad sub-set that excludes QL11, QL12, QL23, & QL24.

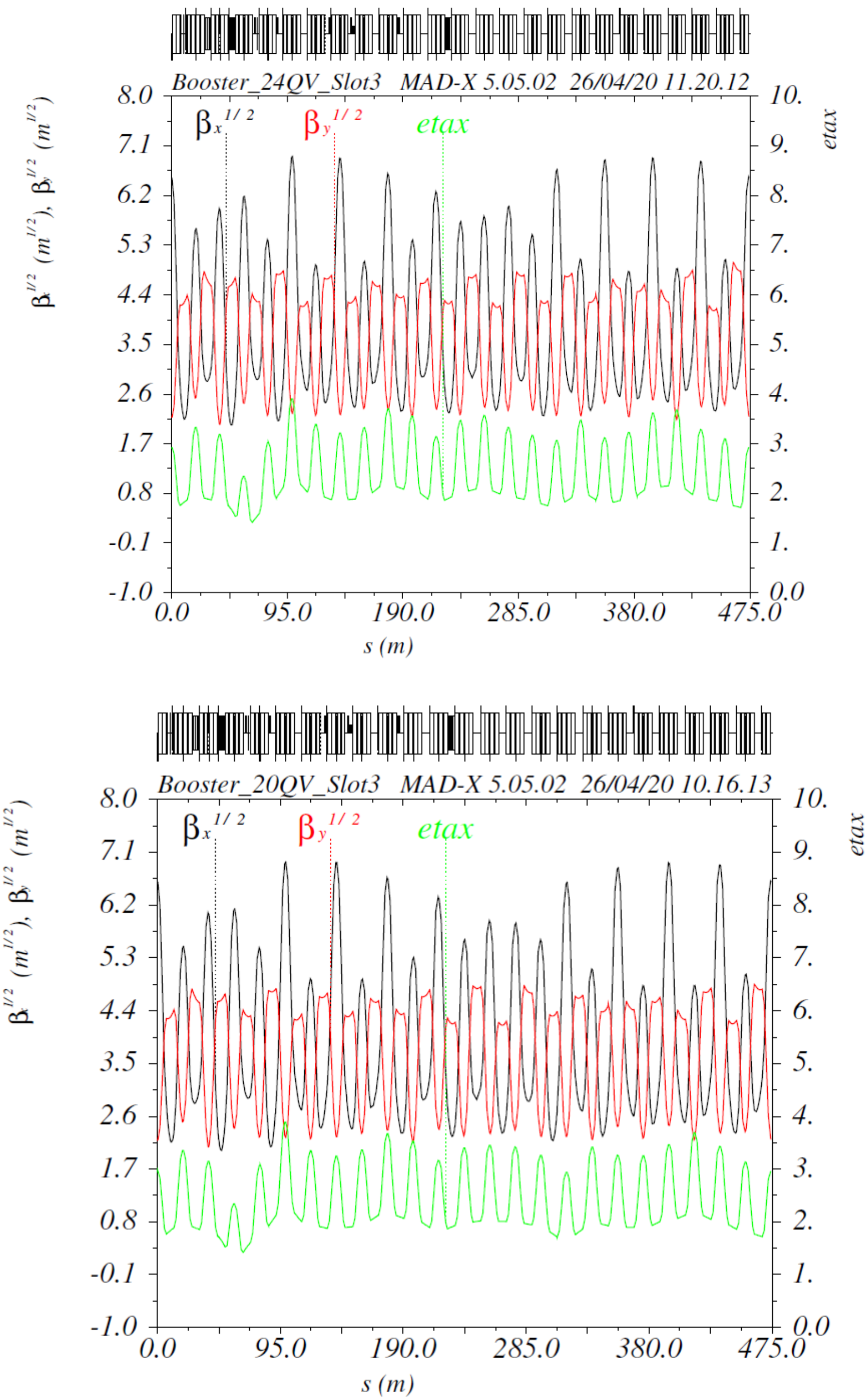
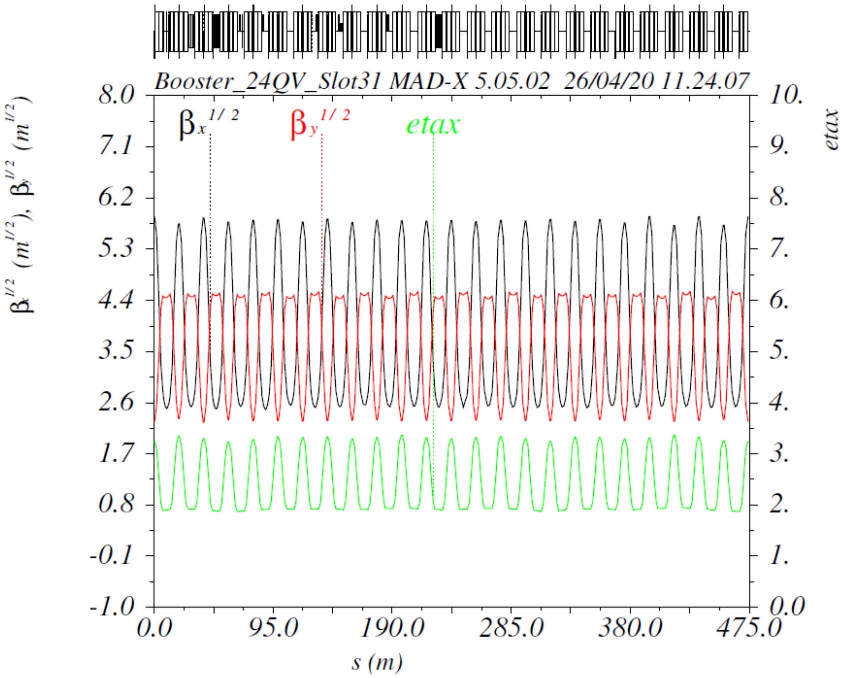


Figure 3. Lattice functions for 24 (top) & 20 (bottom) vertical quad sets at timeslot = 3, and tunes Q1=6.5500, Q2=6.8475.

.



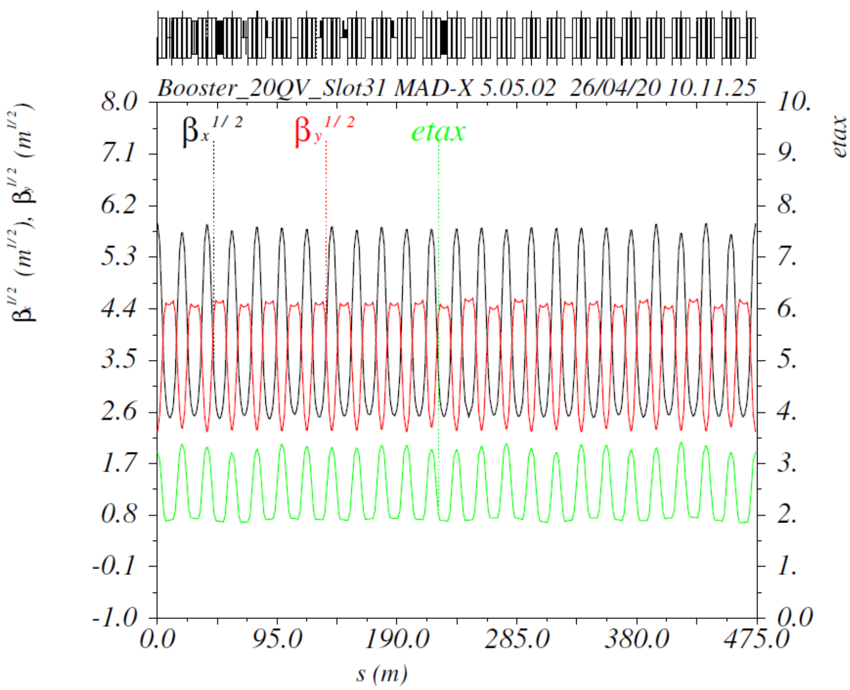


Figure 4. Lattice functions for 24 (top) & 20 (bottom) vertical quad sets at timeslot = 31, and tunes Q1=6.5500, Q2=6.8475.