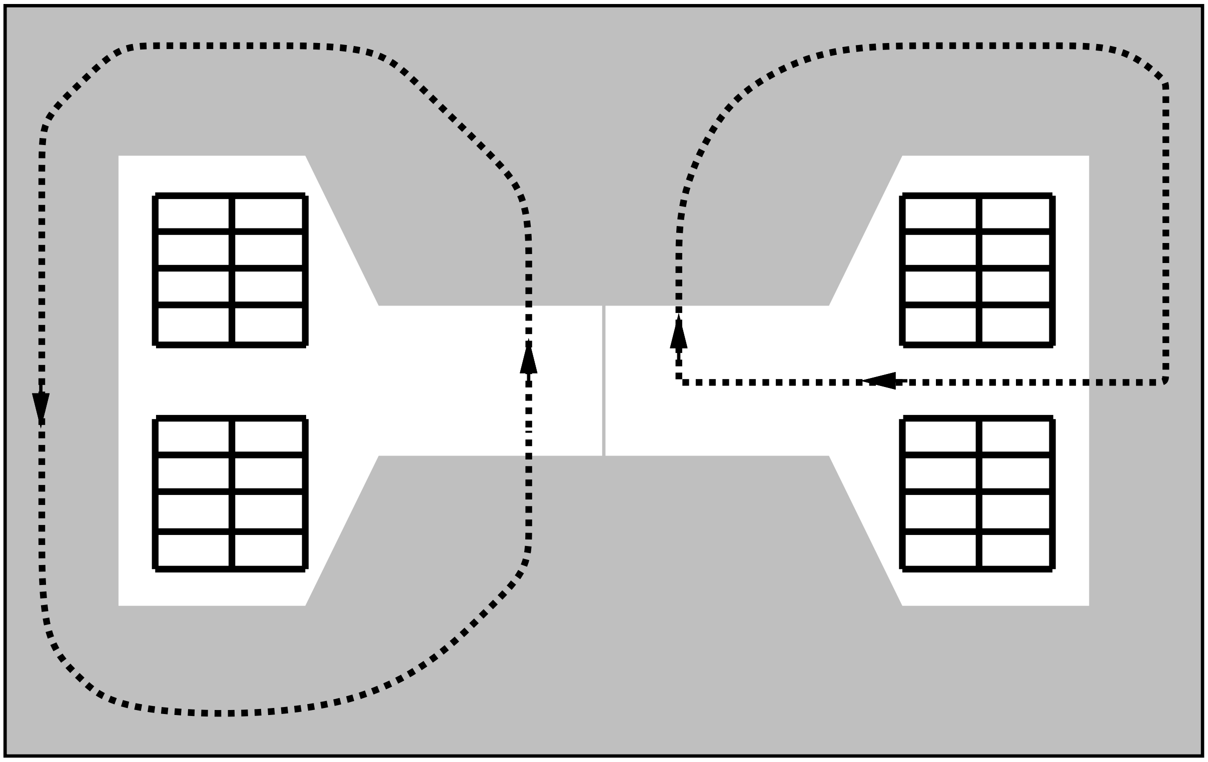
Analytic Form for Fitting Hysteretic Magnet Strength

Bruce C. Brown
IMMW XI at Brookhaven
21-September-1999

At IMMW X at Fermilab, I presented a strategy for studying hysteresis effects in accelerator and beam line magnets. I would like to update that with a report on my progress at finding an analytic form which will fit this data to a precision of 3×10^{-4} or so.

A status report on this work was presented at PAC99 and can be found on my WWW site:
http://www-ap.fnal.gov/ bcbrown/Docs/p-Conf-99-096.ps



To predict magnetic fields we employ Ampere's Law:

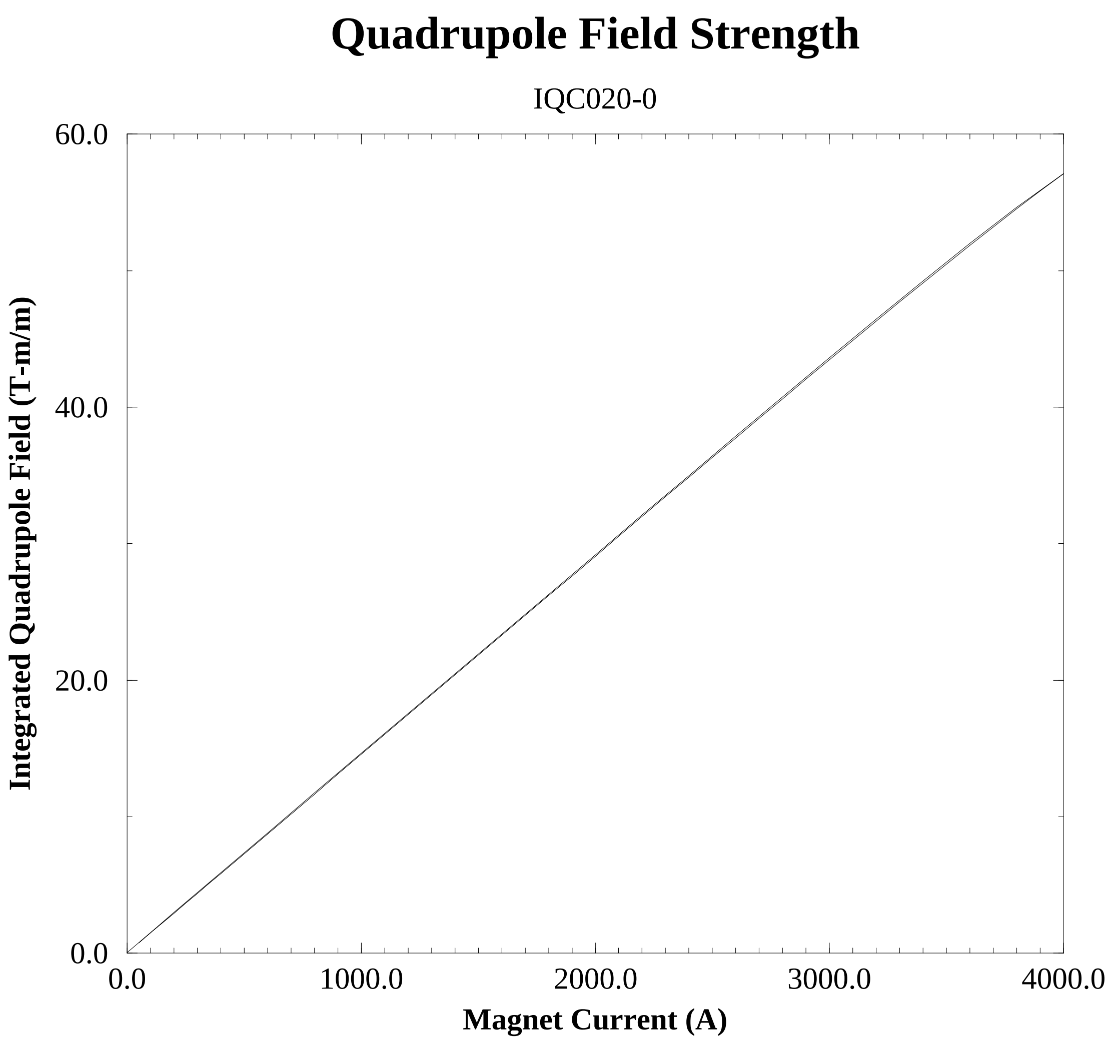
$$\int_g \frac{1}{\mu_0} \vec{B}_g \cdot d\vec{\ell} + \int_{\mathcal{L}} \vec{H} \cdot d\vec{\ell} = N_g I, \qquad (1)$$

where g represents the path in the air gap and $\mathcal{L}$ represents the path through the steel. We will use N_g turns per gap as for loop on left.

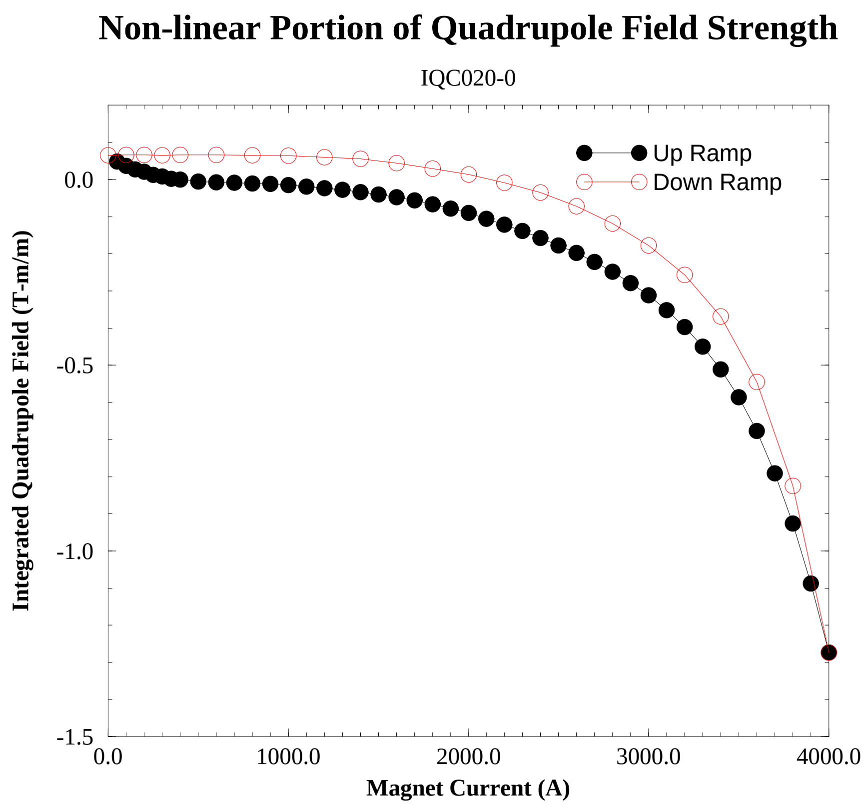
In a well designed multipole magnet (dipole, quadrupole....) the the field in the gap is well represented by the dominant multipole component. We integrate along a field line in the gap (to pole radius A). To be ready to fit integrated field measurements, we integrate along the beam path by multiplying our body field strength by an effective length, L_{eff}. For the integral over the path in steel we choose a typical path along a flux line.

$$B_N L_{eff} = \frac{\mu_0 N N_g L_{eff} I}{2A^N} - \frac{N \mathcal{L} L_{eff}}{2A^N} \mu_0 < H_{steel} > . \tag{2}$$

where N is the harmonic number (1 for dipole), N_g is the number of turns per gap in the coil, A the pole tip radius ($g/2$ for a dipole), $\mathcal{L}$ is the length of a flux line in iron with average H along the path of $<H_{steel}>$. I is the current through the coil. We note that the first term is proportional to I and it represents the field created in idealized iron by the magnet current. The second term describes the field lost in driving the iron. All saturation and hysteretic terms due to iron remanence are described by $<H_{steel}>$.

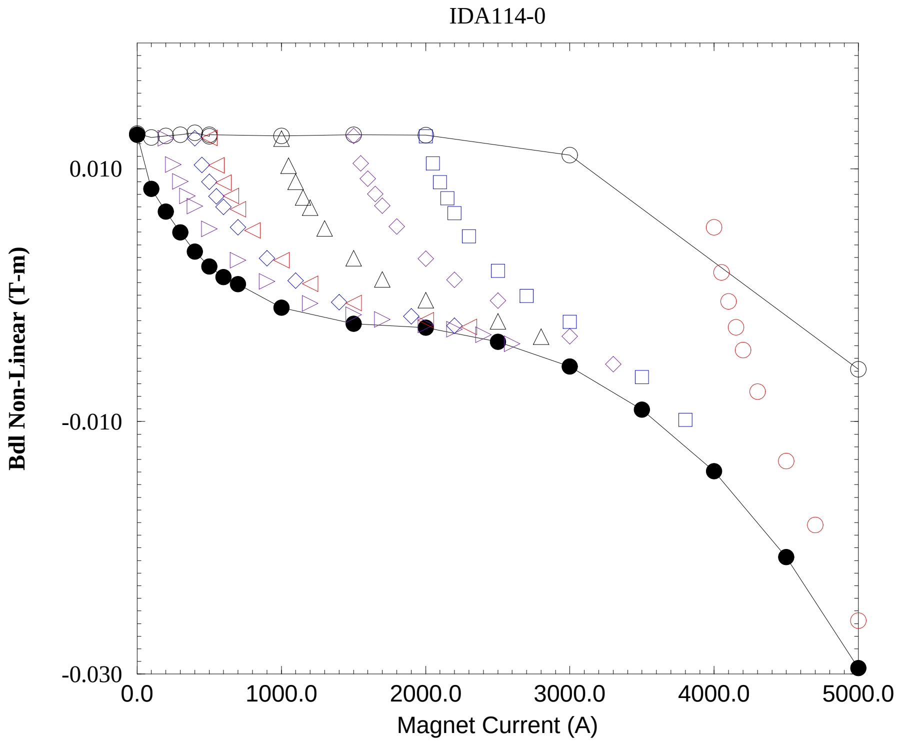


Let us look at some typical data for magnet strength. Note that there is both an up ramp and a down ramp measurement on this plot.

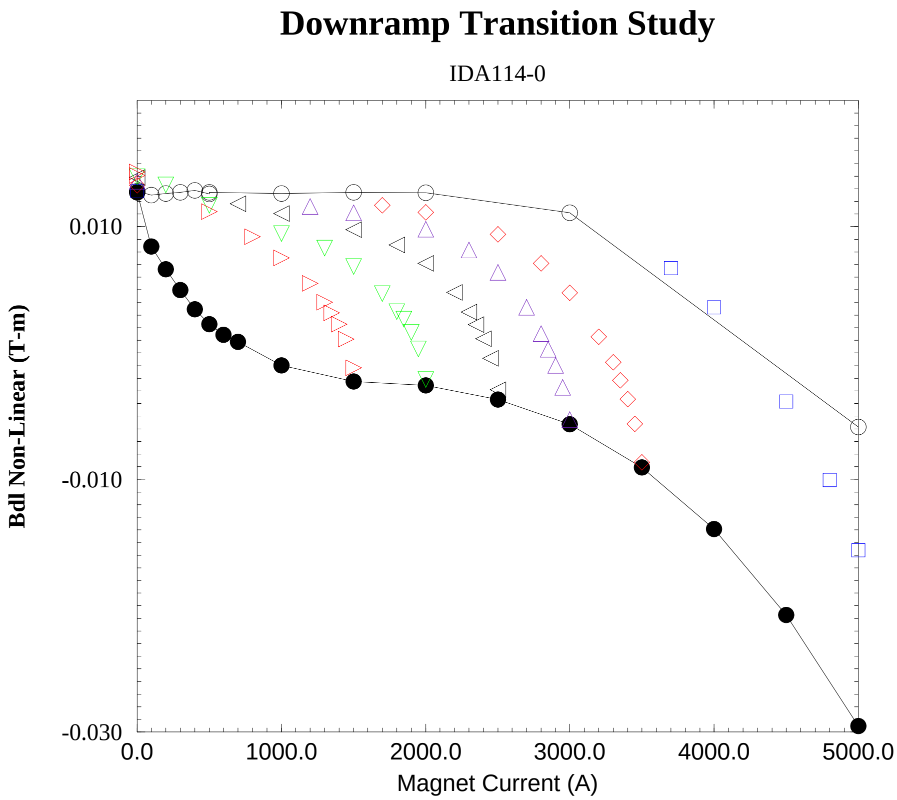


To see the effects of $<H_{steel}>$, we subtract a term linear in current. It can be from fitting the previous plot or by calculation from pole geometry and the number of turns. We see two sorts of contributions in this plot. The upramp strength is less than the downramp strength (hysteresis) and there is a sharp change at high field (saturation).

Downramp Transition Study



Before selecting an analytic fitting form, we need to examine additional measurements which will guide our choices. If we perform measurements with a series of different minimum (reset) fields we find a family of similar curves for the non-linear field.



The results for measurements with various peak excitations again have an obvious pattern with shapes very suggestive of the same form.

Our analytic form will need two types of terms. We have chosen to call the form reached well after current changes as the 'hysteresis curve'. There is an upramp hysteresis curve and a downramp hysteresis curve. We initially note that it has some obvious similarity to a hyperbola which has been suitably rotated and offset.

$$H(I, D) = -\sqrt{h_2}x - \sqrt{h_2 x^2 + h_0}$$

The curves which transitions the strength between the upramp curve and the downramp curve we call Interjacent curves. The exponential character of these is apparent to the most casual observer.

$$J(I, I_r, I_p, D) = A(I_r, I_p, D) \quad e^{-(\frac{I-I_r}{I_{C,D}})}$$

Adding a parabola to provide a little freedom for fitting, we applied this to the data and achieved a fit precision of about 0.3% (30×10^{-4}). We are using this prescription, however inadequate, for Main Injector operation at this time.

To fit the data more precisely, we had to overcome a number of problems:

- The remanent field has a weak dependent on the peak of the last ramp. This is likely to be unimportant for ramps of operational interest, but in trying to get sufficient range of data to constrain the fit parameters, we get enough differences to make this significant.
- The hyperbola is not sufficiently 'rich' to represent the hysteresis curves.
- A single exponential falls too quickly to represent the data.
- The current control was very good (a 10 kA system operating at 500 A gave an RMS magnet strength deviation consistent with less than 20 mA RMS current deviation) but the current readback was about one order of magnitude worse. We 'calibrate' the the control current to get information for fitting.

We consider the magnet strength M ($\int B_1 dl, \int B_2 dl$ or $\int B_3 dl$) to be comprised of four terms, L (linear), R (remanent), H (hysteretic) and J (interjacent) . We continue to explore suitable expressions for these contributions but find useful fits with the following functional relations:

$$M(I, I_r, I_p, D) = \\ L(I) + R(I_p, D) + H(I, D) + J(I, I_r, I_p, D)$$

where I is the magnet current during the measurement, I_r is the reset current (current at last sign change in dI/dt), I_p is the preset current (reset current of last ramp), and D is the ramp direction with +1 for upramps and -1 for downramps. We express the relations with normalized variables to provide consistency of representation among magnets. Use I_{scale} as a maximum current of interest (rounded) and I_S as a characteristic current for saturation.

$$x = \frac{I - I_S}{I_{scale}} \quad x_0 = \frac{-I_S}{I_{scale}}$$

Expressions used for these terms are

$$L(I) = Slope * I$$

$$R(I_p, D) = RemStr_D + RemSlp_D * (I_p - I_{scale})$$

$$\begin{aligned} H(I, D) = & \\ & C_1 * \frac{I}{I_{scale}} \\ & - \sqrt[4]{h_4} x - \sqrt[4]{h_4 x^4 + h_3 x^3 + h_2 x^2 + h_1 x + h_0} \\ & + \sqrt[4]{h_4} x_0 + \sqrt[4]{h_4 x_0^4 + h_3 x_0^3 + h_2 x_0^2 + h_1 x_0 + h_0}. \end{aligned}$$

Note that H is defined to have the value 0 at $I = 0$. Each parameter is distinct for the upramp or downramp curve and could be expressed as h_{iD} or C_{1D} .

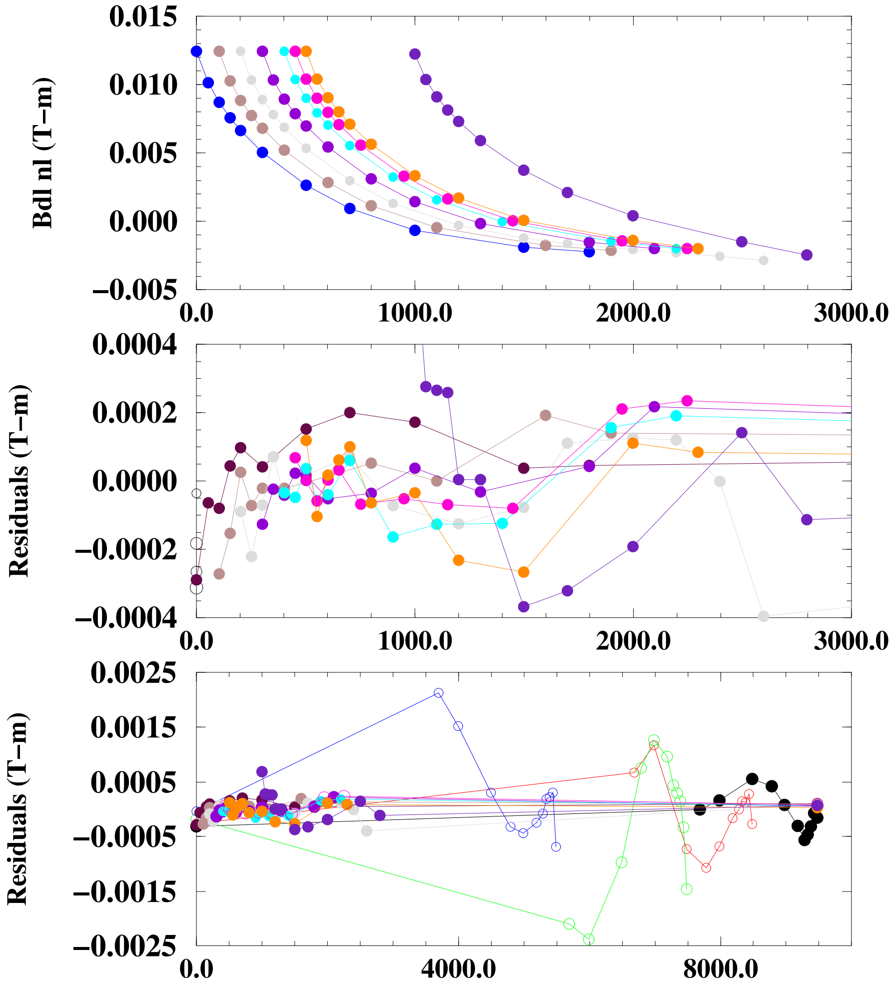
Two forms have been used for fitting J:

$$J(I, I_r, I_p, D) = A(I_r, I_p, D)(se^{-\frac{I-I_r}{I_{C1,D}}} + (1-s)e^{-\frac{I-I_r}{I_{C2,D}}})$$

$$J(I, I_r, I_p, D) = A(I_r, I_p, D)e^{-(\frac{I-I_r}{I_{C,D}})^N}$$

where N is a real number, typically less than 1. The amplitude function A is the difference in hysteresis curves at the reset current.

$$A(I_r, I_p, D) =$$
$$H(I_r, -D) - H(I_r, D) + R(I_p, -D) - R(I_p, D).$$



Selected data from the IDA114-0 hysteresis study were fit with the interjacent curve described by 2 exponentials. Top plot shows fits to the selected upramp data. Center and lower plots show residuals (measured - fitted) on scales which emphasize the low field and high field results.

Summary

- Strength measurements of accelerator magnets, while dominated by the linear strength term have important field components which are not linear in excitation current. These non-linear terms have surprisingly simple regularities which permit analytic descriptions.
- To good accuracy, these non-linear terms exponentially approach a common hysteresis curve following a sign change in dI/dt. A small effect due to the reset (or preset) current may remain.
- The Interjacent curves which characterize the fashion in which the strength approaches the hysteresis curve is nearly exponential. Fits using two exponentials or a modified exponential are sufficient for present requirements.
- Analytic fitting functions have been found which describe these effects well enough to leave fitting residuals which are less than 5×10^{-4} relative to the magnet strength at each current.
- Data have been measured on six or more magnet designs. The same characteristics are apparent in all of them. Efforts to get a complete software system which will fit all of this measured data is continuing.