Monthly Group Report

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Monthly Group Meeting
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Fermilab

Beams-doc-8699 : Evolution of H Species Angular Distributions from Multiple Scattering in Thin Stripping Foils.

In stripping of H^- the H species undergo extensive atomic scattering reactions as they traverse the foil which leads to emittance growth in the final beam. This note derives equations for round beams that sum the infinite multiple scattering series exactly for high energy incident H beams and small scattering angles.

$$\frac{dH^-(\theta,z)}{dz} = -(\sigma_{0-} + \sigma_{+-}) \cdot H^-(\theta,z) + \left[\int d\Omega' \frac{d\sigma_{--}}{d\Omega'} \cdot H^-(\theta'',z) - \sigma_{--} \cdot H^-(\theta,z)\right]$$

$$\frac{dH^0(\theta,z)}{dz} = -\sigma_{+0} \cdot H^0(\theta,z) + \int d\Omega' \frac{d\sigma_{0-}}{d\Omega'} H^-(\theta'',z) + \left[\int d\Omega' \frac{d\sigma_{00}}{d\Omega'} H^0(\theta'',z) - \sigma_{00} \cdot H^0(\theta,z)\right]$$

$$\frac{dH^+(\theta,z)}{dz} = +\int d\Omega' \frac{d\sigma_{+0}}{d\Omega'} H^0(\theta'',z) + \int d\Omega' \frac{d\sigma_{+-}}{d\Omega'} H^-(\theta'',z) + \left[\int d\Omega' \frac{d\sigma_{++}}{d\Omega'} H^+(\theta'',z) - \sigma_{++} \cdot H^+(\theta,z)\right]$$

There are 4 steps to finding the solutions:

1. The equations are recast in terms of Hankel transforms of the *H* species;
2. The resulting distributions are rewritten using Graf's summation expansion of Bessel functions. This step forms the crux of solving for the distributions because integration over the azimuthal angle φ leads to expressions separable in the angular variables;
3. The result of step 2. is 3 coupled linear differential equations which can be solved analytically, and;
4. *H* species angular distributions are the inverse Hankel transforms of step 3.

$$h(k,z) = \int_0^\infty \theta d\theta J_0(k\theta) H(\theta,z)$$

$$\int_0^\infty \omega d\omega \; J_\nu(x\omega) J_\nu(x'\omega) = \frac{\delta(x-x')}{x}$$

$$H(\theta,z) = \int_0^\infty k dk J_0(k\theta) h(k,z)$$

$$J_0\left(k\sqrt{\theta^2+\theta'^2-2\theta\theta'\cdot\cos(\varphi)}\right)=J_0(k\theta)J_0(k\theta')+2\sum_{n=1}^{\infty}J_n(k\theta)J_n(k\theta')\cdot\cos(n\varphi)$$

$$\boxed{\therefore\int_0^{2\pi}d\varphi\cdot J_0\left(k'\left|\vec{\theta}-\vec{\theta}'\right|\right)=2\pi\cdot J_0(k'\theta)J_0(k'\theta'):\quad \text{i.e: separable in } \theta,\theta'}$$

$$\frac{dh^-(k,z)}{dz}=-[(\sigma_{0-}+\sigma_{+-})-Q_{--}(k)]\,h^-(k,z)$$

$$\frac{dh^0(k,z)}{dz}=-[\sigma_{+0}-Q_{00}(k)]\,h^o(k,z)+Q_{0-}(k)\,h^-(k,z)$$

$$\frac{dh^+(k,z)}{dz}=Q_{++}(k)h^+(k,z)+Q_{+0}(k)h^0(k,z)+Q_{+-}(k)h^-(k,z)$$

$$Q_{\beta\alpha}(k)\equiv\int_0^{\infty}d\theta'\frac{d\sigma_{\beta\alpha}}{d\theta'}\cdot\left[J_0(k\theta')-\delta_{\beta\alpha}\right]$$

$$Q_{\beta\alpha}(0)=\sigma_{\beta\alpha}\cdot\left[1-\delta_{\beta\alpha}\right]$$

$$h^{-}(k,z) = h^{-}(k,0)e^{-[(\sigma_{0-}+\sigma_{+-})-Q_{--}]z}$$

$$h^{0}(k,z) = h^{-}(k,0)\,\frac{Q_{0-}}{\left[\left((\sigma_{0-}+\sigma_{+-})-Q_{--}\right)-(\sigma_{+0}-Q_{00})\right]}\left[e^{-(\sigma_{+0}-Q_{00})z} - e^{-[(\sigma_{0-}+\sigma_{+-})-Q_{--}]z}\right]$$

$$h^{+}(k,z) = h^{-}(k,0)\left\{\frac{Q_{+0}Q_{0-}}{\left[\left((\sigma_{0-}+\sigma_{+-})-Q_{--}\right)-\left(\sigma_{+0}-Q_{00}\right)\right]}\right.$$
$$\cdot\left[\frac{\left[e^{Q_{++}\cdot z} - e^{-(\sigma_{+0}-Q_{00})z}\right]}{[\sigma_{+0}-Q_{00}+Q_{++}]} - \frac{\left[e^{Q_{++}\cdot z} - e^{-[(\sigma_{0-}+\sigma_{+-})-Q_{--}]z}\right]}{[(\sigma_{0-}+\sigma_{+-})-Q_{--}+Q_{++}]}\right] + Q_{+-}$$
$$\left.\cdot\frac{\left[e^{Q_{++}\cdot z} - e^{-[(\sigma_{0-}+\sigma_{+-})-Q_{--}]z}\right]}{[(\sigma_{0-}+\sigma_{+-})-Q_{--}+Q_{++}]}\right\}$$

$$\boxed{H^{-,0,+}(\theta,z) = \int_0^{\infty} k\,dk\,J_0(k\theta)\cdot h^{-,0,+}(k,z)}$$

Next

- To implement the previous eqn's I need:
  - a Fortran compiler (and a plotting package would be nice too)
  - elastic & inelastic carbon atomic form factors

- Currently looking into generalizing these results to cover elliptical angular distributions (in Booster at the foil $\beta_Y/\beta_X \approx 3$). (This is proving to be a bigger headache than anticipated).

- I have an idea percolating in my brain thingy how it might be possible to raise Booster γ_t above ~9.5.