Short-range wake fields in plasma acceleration

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In collaboration with Valeri Lebedev and Alexey Burov (Fermilab) and
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Aug 14, 2018

Acknowledgements

- We would like to thank our UCLA (Weiming An) and SLAC colleagues for fruitful discussions and computer simulations.

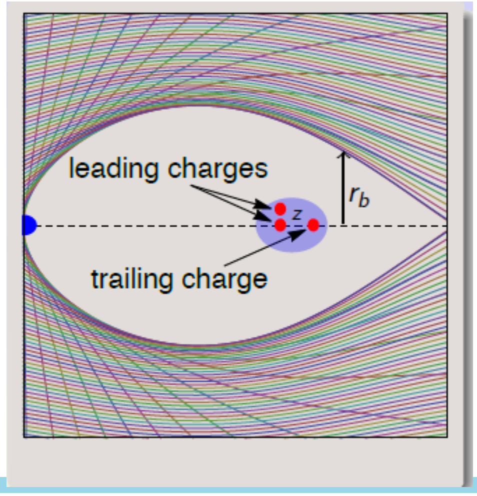
- Some of the results presented in this talk are based on our recent publication, “Efficiency versus instability in plasma accelerators”, Phys. Rev. Accel. Beams 20, 121301 (2017)

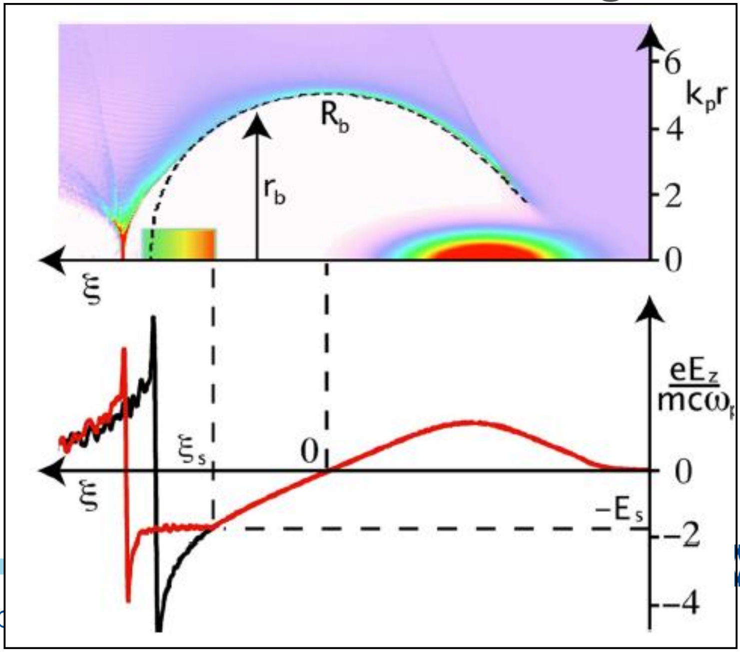
- This work is funded by the DOE HEP and by a DOE SBIR Phase 1 grant

Plasma short-range wakefields

- The terminology of wakefields in plasma can be confusing. The original meaning of the wake in plasma is the field generated by the drive bunch, which accelerates the trailing bunch. (The driver could be particle beam or laser)

- In this presentation, by wakefields I mean the fields (longitudinal and transverse) with which the trailing bunch acts on itself.





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Transverse beam break-up (BBU) instability

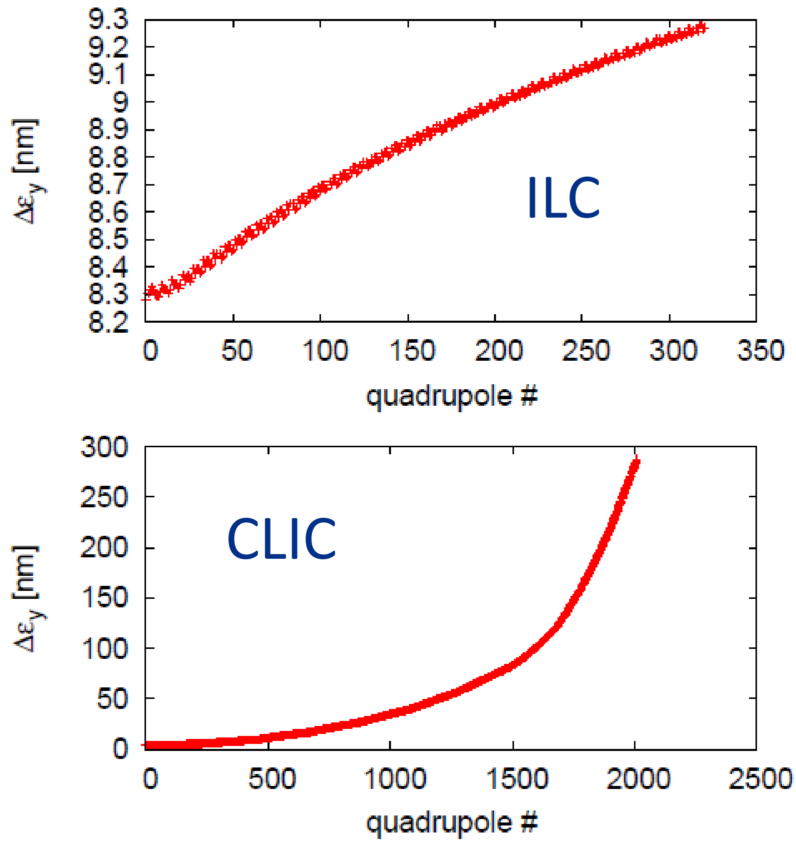
- Transverse wakes act as deflecting force on bunch tail
  - beam position jitter is exponentially amplified

Beam Stability

- Transverse stability of a beam with initial offset of σ_y
  - no energy spread assumed in the beam
  - emittance with respect to the beam axis is shown

$\Rightarrow$ acceptable for ILC (top)

$\Rightarrow$ would be intolerable for CLIC (bottom)



D. Schulte, 6th Linear Collider School 2011, Main Linac Basics 69

Short-range transverse wake (for solid walls)

$$W_{\perp}(z) = \frac{8z}{a^4}$$

$a \approx 35$ mm (ILC)

$a \approx 3.5$ mm (CLIC)

What about plasma?

$a \sim 0.1$ mm (PWFA)

BBU illustration

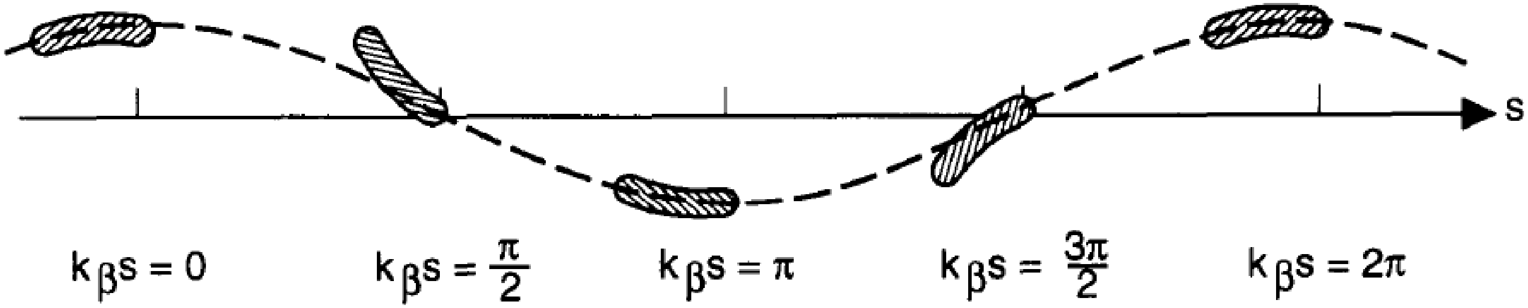


Figure 3.3. Sequence of snapshots of a beam undergoing dipole beam breakup instability in a linac. Values of $k_\beta s$ indicated are modulo 2π. The dashed curves indicate the trajectory of the bunch head.

- See A. Chao, "Physics of collective beam instabilities in high energy accelerators (Wiley, 1993)."
- The growth rate is determined by the ratio of defocusing (wake) force to focusing force (the wake parameter):

$$\eta_t = -\frac{F_t}{F_r} = \frac{e^2 r}{F_r(r)} \int_0^L \frac{dN}{d\xi} W_\perp(\xi) d\xi$$

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The BBU instability development

$$\frac{d^2X(\mu,\xi)}{d\mu^2}+X(\mu,\xi)=\frac{2\eta_t}{L_t^2}\int_0^{\xi}X(\mu,\xi')(\xi-\xi')d\xi'.$$

$$X=\frac{x}{\sqrt{\beta}}\sqrt{\frac{p}{p_0}};\quad \beta=k_p^{-1}\sqrt{2\gamma}$$

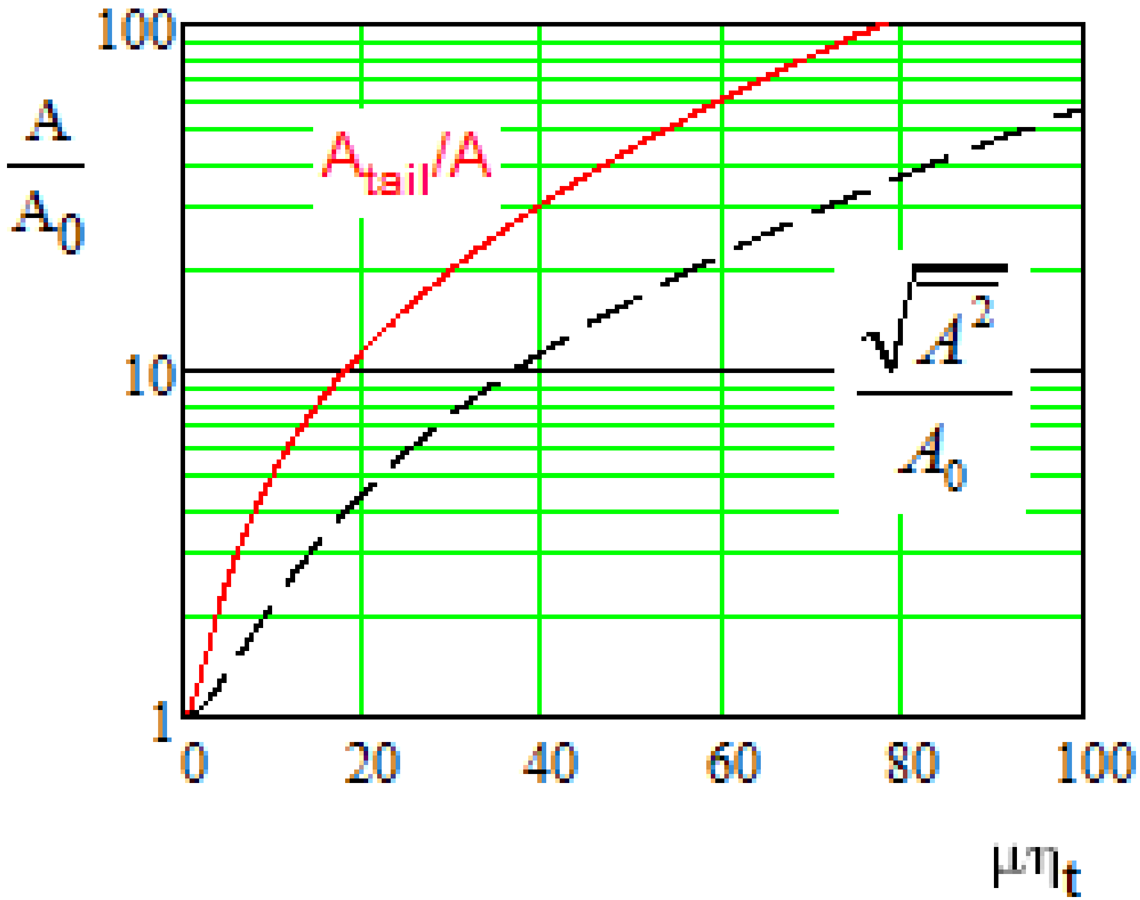
$$d\mu=k_p ds/\sqrt{2\gamma}$$

μ -- betatron phase advance

- For $\eta_t \ll 1$ and $\Delta p/p=0$ it was solved in:
  - C. B. Schroeder, D. H. Whittum, and J. S. Wurtele, “Multimode Analysis of the Hollow Plasma Channel Wakefield Accelerator”, Phys. Rev. Lett. **82**, n.6, 1999, pp. 1177-1180.
- Approximate solutions (it’s a very good fit, <10% deviation):

$$\frac{A}{A_0}=\exp\left(\frac{(\mu\eta_t)^2}{10+1.4(\mu\eta_t)^{1.57}}\right);\quad \begin{matrix}\mu\eta_t\le 100\\ \eta_t\le 0.1\end{matrix}$$

$$\frac{\sqrt{\overline{A^2}}}{A_0}=\exp\left(\frac{(\mu\eta_t)^2}{60+2.2(\mu\eta_t)^{1.57}}\right);\quad \begin{matrix}\mu\eta_t\le 100\\ \eta_t\le 0.1\end{matrix}$$



- Note that *A* is a normalized particle amplitude. For a constant plasma density and without instability *A* would stay constant, while the initial physical amplitude *x* should decrease as $1/\gamma^{\frac{1}{4}}$

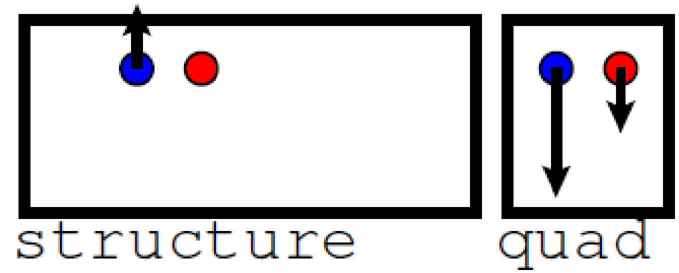
Beam breakup in various collider concepts

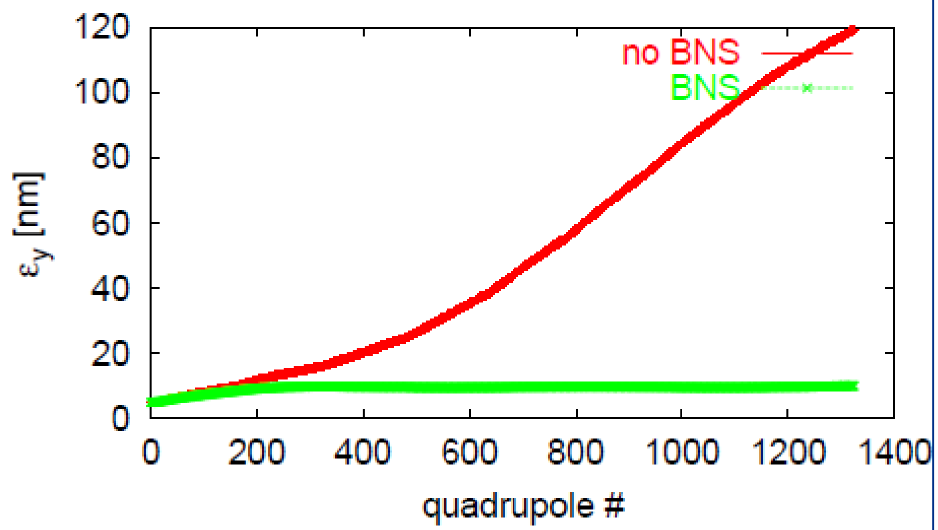
- ILC
  - Not important; bunch rf phase is selected to compensate for long wake and to minimize the momentum spread
- CLIC
  - Important; bunch rf phase is selected to introduce an energy chirp along the bunch for BNS damping (~0.5% rms). May need to be de-chirped after acceleration to meet final-focus energy acceptance requirements
- PWFA – subject of our study
  - Critical;

CLIC strategy: BNS damping + < µm alignment of cavities

Achieving Beam Stability

- Transverse wakes act as defocusing force on tail
  - ⇒ beam jitter is exponentially amplified
- BNS (Balakin, Novokhatsky, and Smirnov) damping prevents this growth
  - manipulate RF phases to have energy spread
  - take spread out at end





D. Schulte, 6th Linear Collider School 2011, Main Linac Basics 70

Strategy was also used at the SLC…

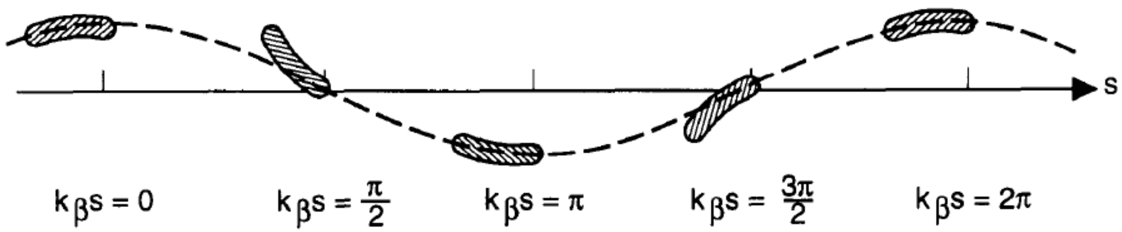


Figure 3.3. Sequence of snapshots of a beam undergoing dipole beam breakup instability in a linac. Values of $k_\beta s$ indicated are modulo 2π. The dashed curves indicate the trajectory of the bunch head.

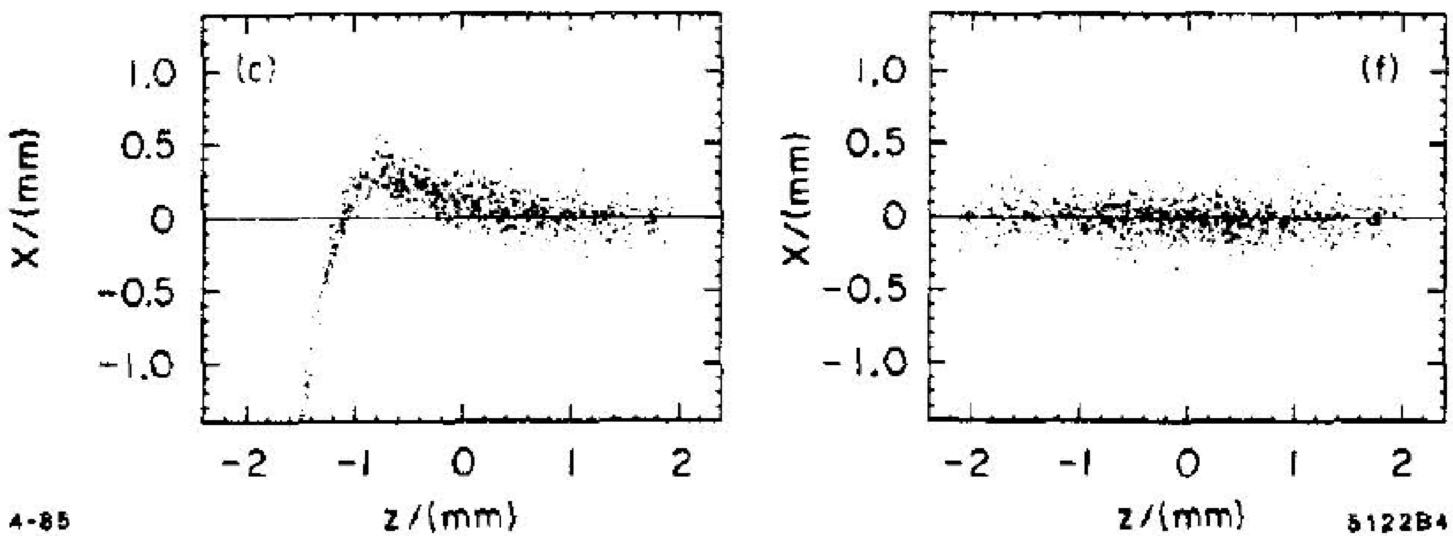


Figure 34: Multiparticle simulation of a particle bunch passing through the SLAC linac without (left) and with BNS damping (right) [36].

BNS damping: what is it?

- Assume a constant long. density of trailing bunch. Chromatic detuning of tail particles allows to keep amplitudes constant

$$\frac{d^2X(\mu,\xi)}{d\mu^2}+\frac{X(\mu,\xi)}{1+\Delta p(\xi)/p}=\frac{2\eta_t}{L_t^2}\int_0^{\xi}X(\mu,\xi')(\xi-\xi')d\xi'.$$

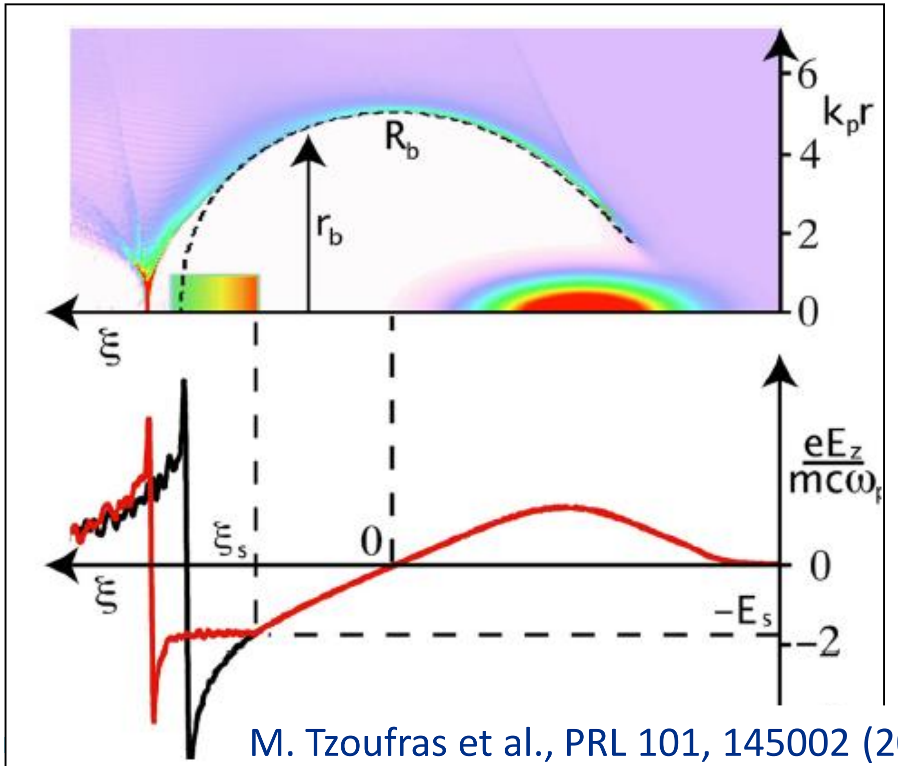
$$\eta_t=-\frac{F_t}{F_r}=\frac{1}{2\pi n_0}\int_0^{L}\frac{dN}{d\xi}W_\perp(\xi)d\xi$$

-- Transverse wake parameter in a PWA blow-out regime

$$X(\xi)=\text{const}\quad\rightarrow\quad\frac{\Delta p(\xi)}{p}=-\eta_t\frac{\xi^2}{L_t^2}$$

Acceleration in a plasma blow-out regime

- The Q-factor is very low (~1) – must accelerate the trailing bunch within the same bubble as the driver!
- Cannot add energy between bunches, thus a single bunch must absorb as much energy as possible from the wake field.



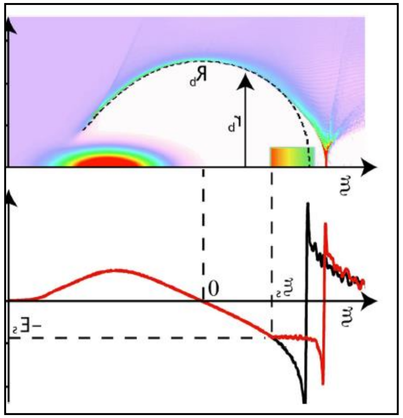
M. Tzoufras et al., PRL 101, 145002 (2008)

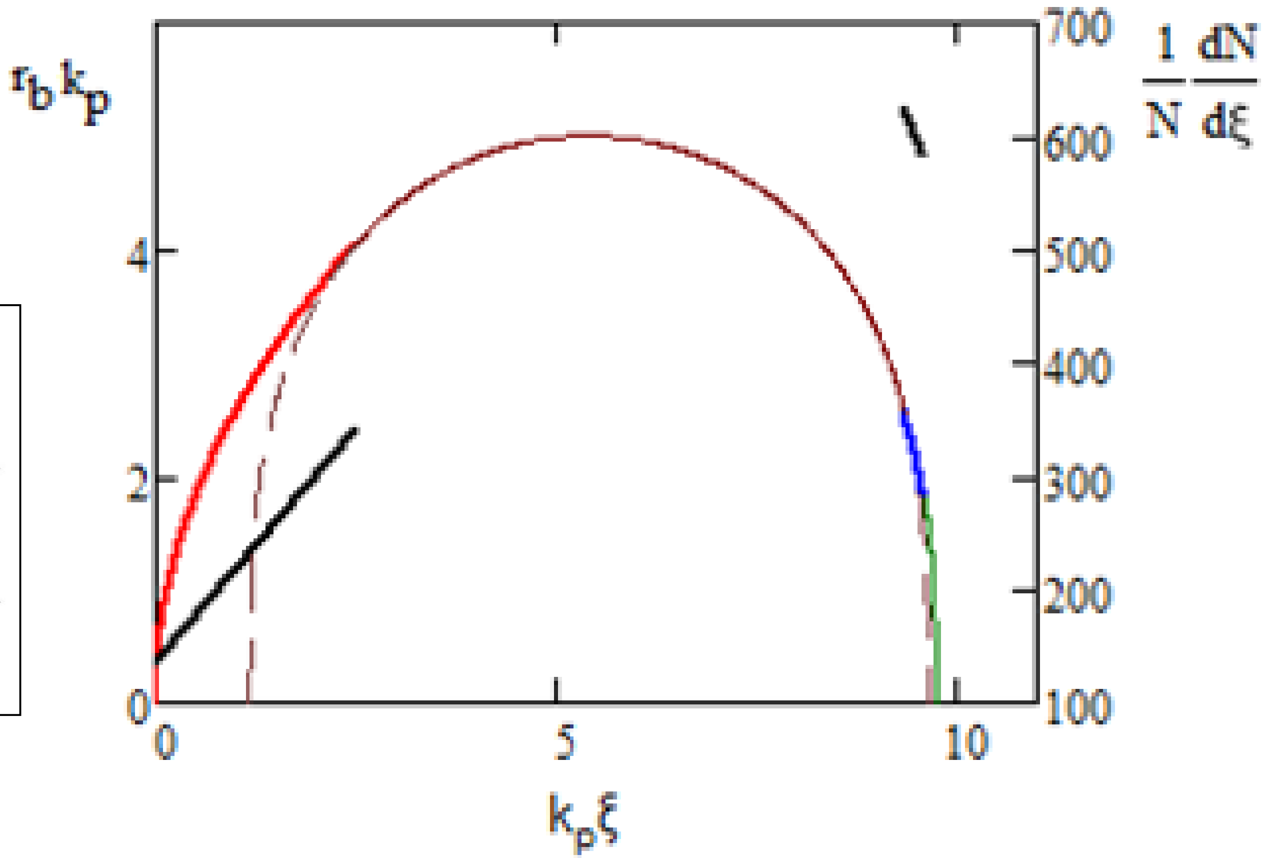
To achieve L ~10^{34}, bunches should have ~10^{10} particles (similar to ILC and CLIC). In principle, we can envision a scheme with fewer particles/bunch and a higher rep rate, but the beam loading still needs to be high for efficiency reasons.

Power transfer from drive to trailing bunches

Trapezoidal charge line density distribution → constant electric field

Example:
50% power transfer efficiency drive to trailing





See M. Tzoufras et al., PRL 101, 145002 (2008)

The power transfer efficiency of 50% and the transformer ratio of 2. For n_0=10^{17} cm^{-3} the drive bunch parameters are chosen to be $R_b k_p$=5, $L_d k_p$=2.5 yielding the decelerating field of E_d = 50 GV/m and N_d=3.55·10^{10}. The trailing bunch parameters are: r_{t2}=0.518R_b, r_{t1}=0.373R_b, E_t = 100 GV/m, N_t=8.86·10^9.

The efficiency-instability relation in a blowout regime

$$\eta_t \approx \frac{{\eta_P}^2}{4\left(1-\eta_P\right)}, \qquad \frac{r_{t2}}{R_b} \le 0.7$$

- This formula does not include any details of beams and plasma, being amazingly universal!
- Note: this formula is an estimate on a "low side". On a "high side", we estimate it as: $\eta_t \approx {\eta_P}^2 / \left(4\left(1-\eta_P\right)^2\right)$
- Example: $\eta_P = 50\% \rightarrow \quad 0.125 < \eta_t < 0.25$

  $\eta_P = 25\% \rightarrow \quad 0.021 < \eta_t < 0.028$

See: "Efficiency versus instability in plasma accelerators", PRAB 20, 121301, 2017

Examples (FACET-II)

Plasma: $n_0 = 4\times10^{16}\ \mathrm{cm}^{-3}$, 60 cm long channel

- p_i=10 GeV/c for both the drive and the trailing bunches, and the final momentum of trailing bunch p_f=21 GeV/c, N_d=1x10^{10} and N_t=4.3x10^9

$$\eta_P = 50\%,\ \eta_t \approx 0.12,\ \mu\eta_t \approx 11.5 \ \rightarrow\ \frac{A}{A_0} \approx 5.8$$

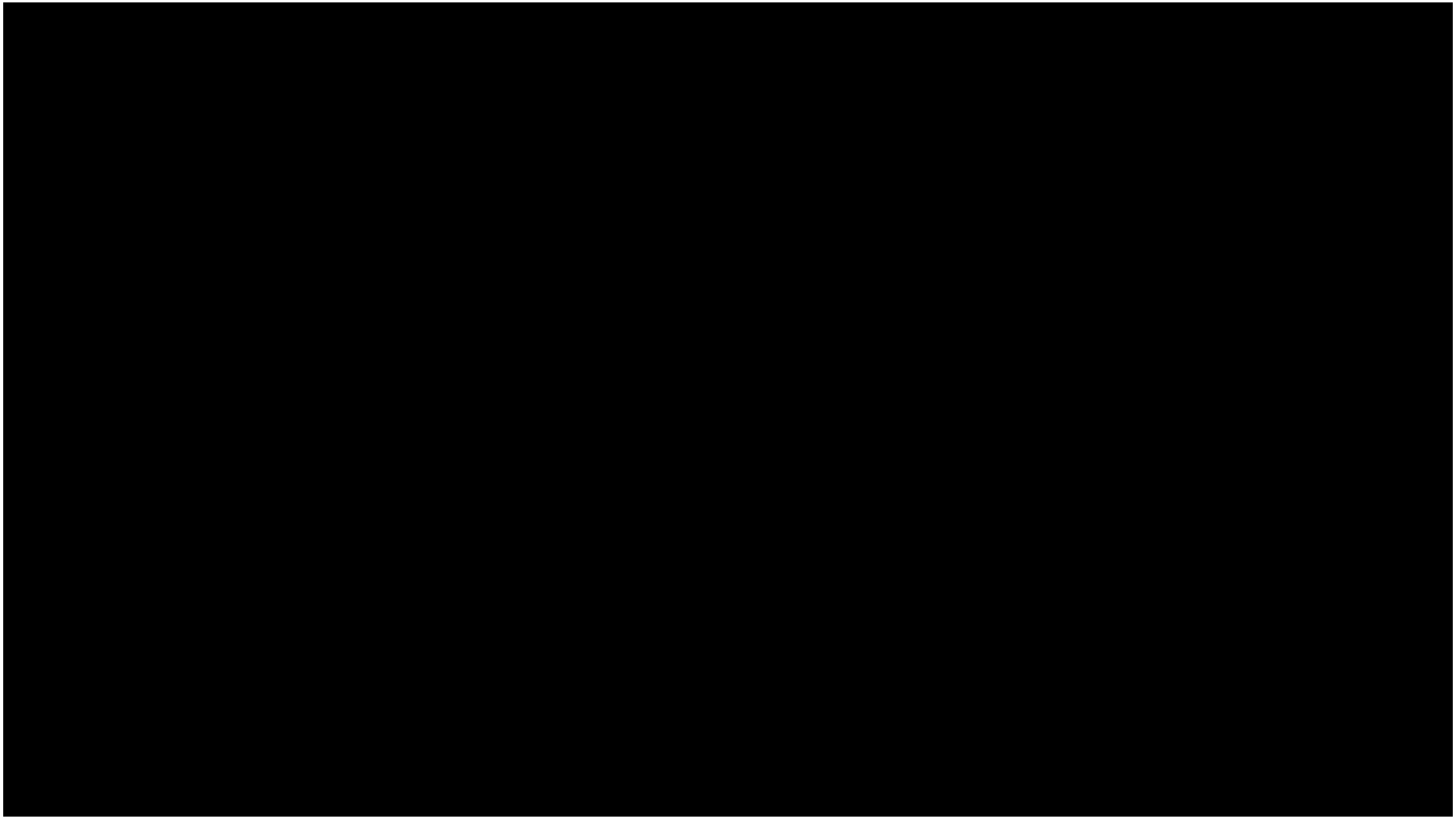
- If one reduces the power efficiency:

$$\eta_P = 25\%,\ \eta_t \approx 0.021,\ \mu\eta_t \approx 2 \ \rightarrow\ \frac{A}{A_0} \approx 1.3$$

- Of course, the final momentum is now p_f=15.5 GeV/c (for the same number of particles)

$$\delta\varepsilon_n = \frac{\delta x^2}{2\beta_i}\gamma_i\left(\frac{\overline{A^2}}{A_0^{\,2}}\right), \quad \beta_i = \frac{\sqrt{2\gamma_i}}{k_p}$$

Case I: ~50% power efficiency $\eta_P = 50\%,\ \eta_t \approx 0.13$



Courtesy of UCLA

Hosing Study for FACET II: Case I

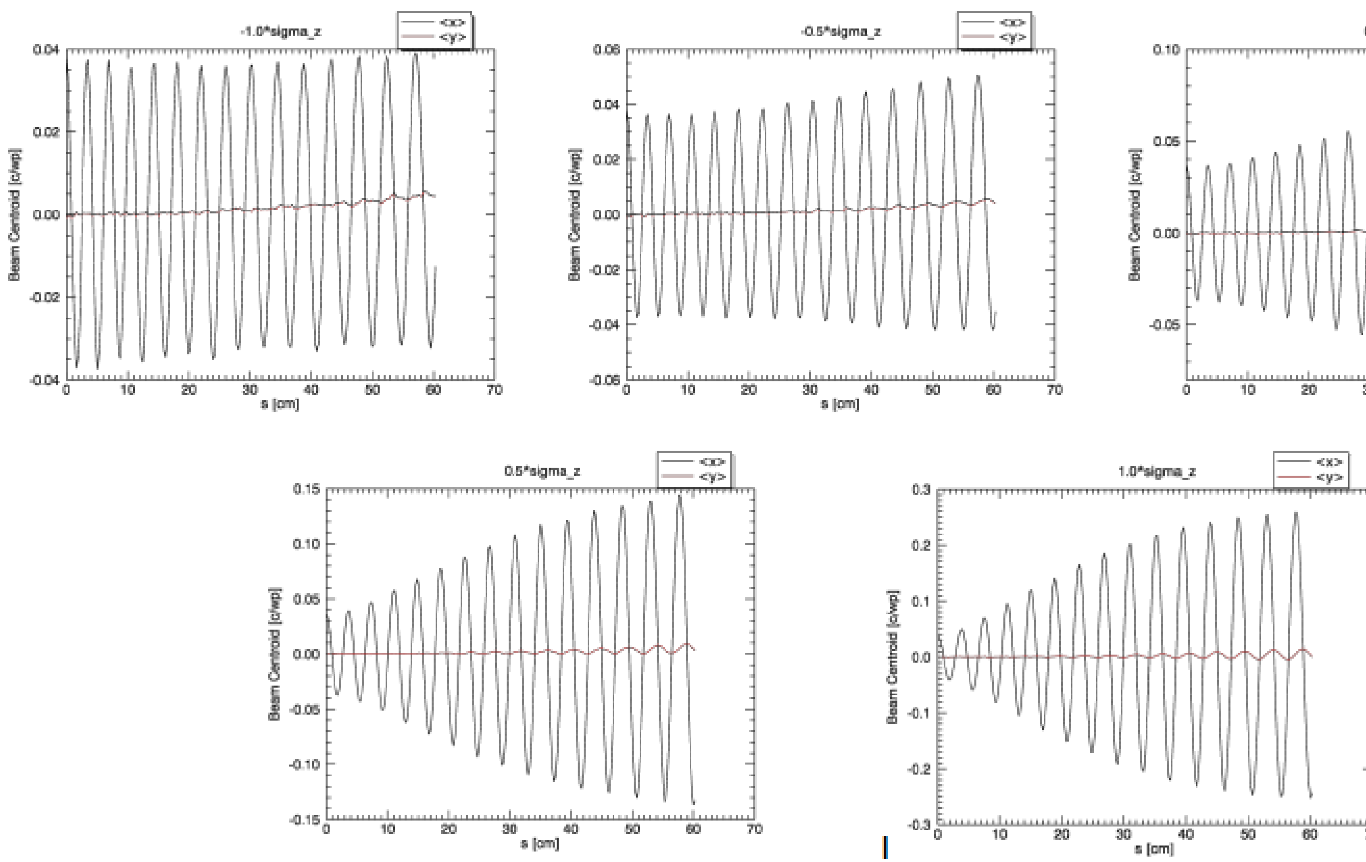
UCLA

Drive Beam: E = 10 GeV, I_{peak}=15 kA
σ_r = 3.65 μm, σ_z = 12.77 μm ,
N =1.0 x 10^{10} (1.6 nC), ε_N = 10 μm

Trailing Beam: E = 10 GeV, I_{peak}=9 kA
σ_r = 3.65 μm, σ_z = 6.38 μm ,
N =4.33 x 10^{9} (0.69 nC), ε_N = 10 μm
(transversely offset by 1 μm)

Distance between two bunches: 150 μm
Plasma Density: 4.0 x 10^{16} cm^{-3}

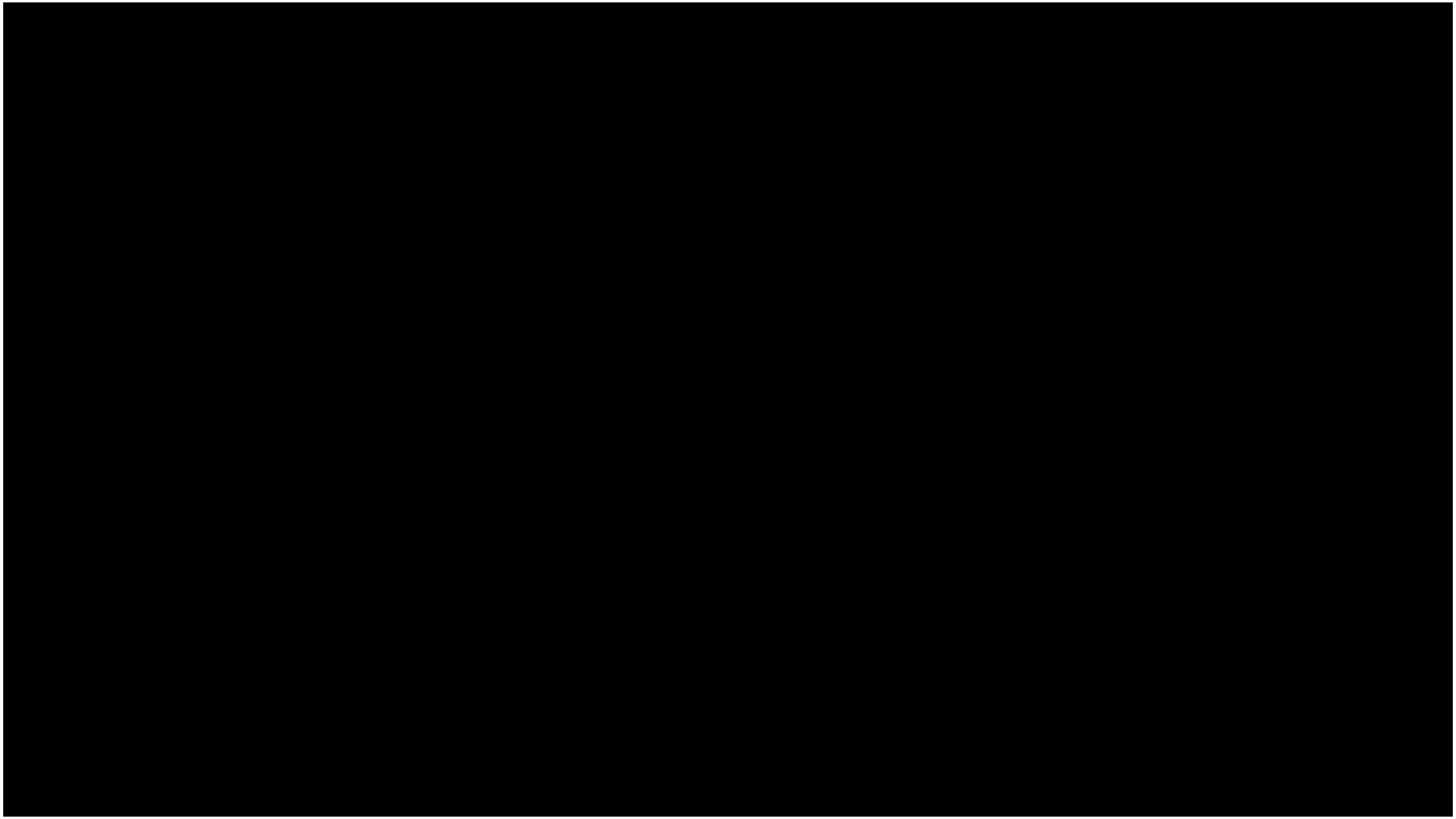
Trailing beam centroid vs s in different slices



50% power efficiency

Case II: ~25% power efficiency

$$\eta_P = 25\%,\ \ \eta_t \approx 0.02$$



Courtesy of UCLA

Hosing Study for FACET II: Case II

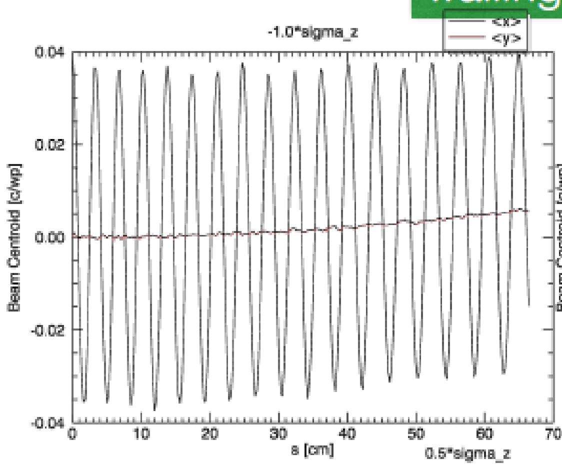
Drive Beam: E = 10 GeV, I_{peak}=15 kA
σ_r = 3.65 μm, σ_z = 12.77 μm ,
N =1.0 x 10^{10} (1.6 nC), ε_N = 10 μm

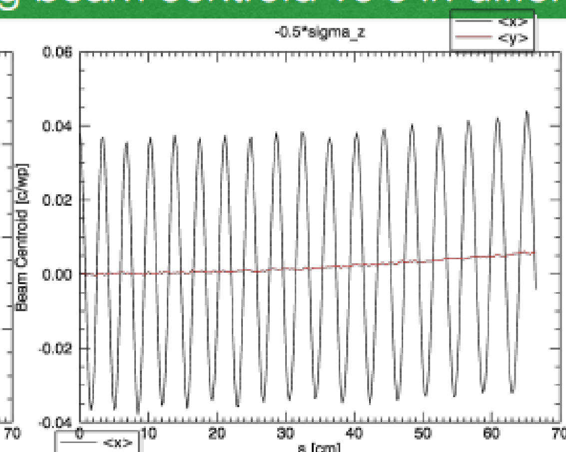
Distance between two bunches: 108 μm
Plasma Density: 4.0 x 10^{16} cm^{-3}

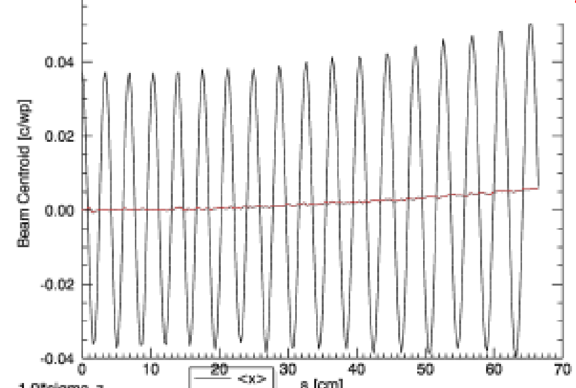
Trailing Beam: E = 10 GeV, I_{peak}=9 kA
σ_r = 3.65 μm, σ_z = 6.38 μm ,
N =4.33 x 10^9 (0.69 nC), ε_N = 10 μm
(transversely offset by 1 um)

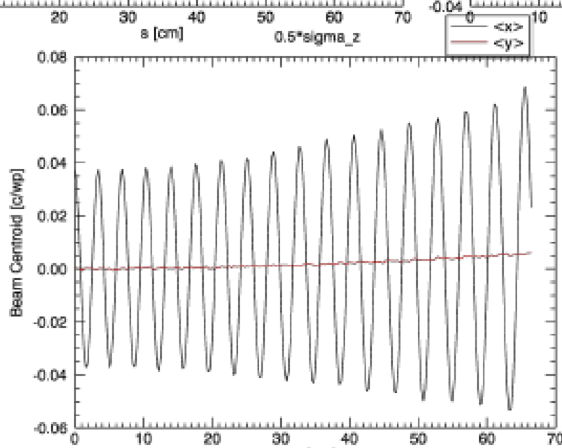
25% power efficiency

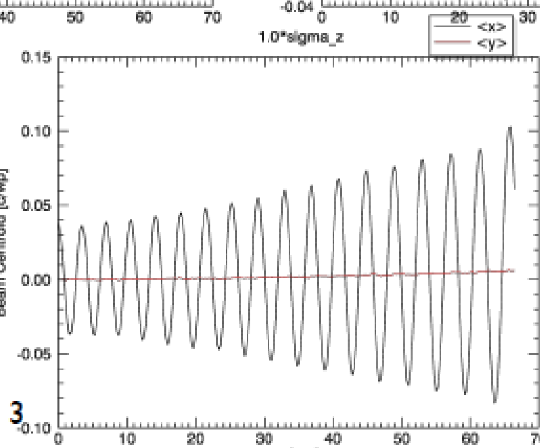
Trailing beam centroid vs s in different slices











Mitigation by momentum chirp (classical BNS)

$$\frac{d^2X}{d\mu^2} + \frac{X}{1+\Delta p/p} = \frac{2\eta_t}{L_t^2}\int_0^{\xi} X(\xi')(\xi-\xi')d\xi'.$$

$$X(\xi) = \text{const} \;\rightarrow\; \frac{\Delta p(\xi)}{p} = -\eta_t \frac{\xi^2}{L_t^2}$$

Goal:

$$\eta_P = 50\%,\;\; \eta_t \approx 0.13$$

- The maximum allowed momentum spread might be determined by the stage-to-stage transition optics
- If one can tolerate $\frac{\Delta p}{p} \le 1\%$ than $\eta_t \le 0.01$

$$\eta_t \approx \frac{\eta_P^{\;2}}{4(1-\eta_P)}$$

CLIC Design: $\frac{\Delta p}{p} \le 0.5\%$

→ Therefore, the max power efficiency is

$$\eta_P \le 18\%$$

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The role of plasma ions

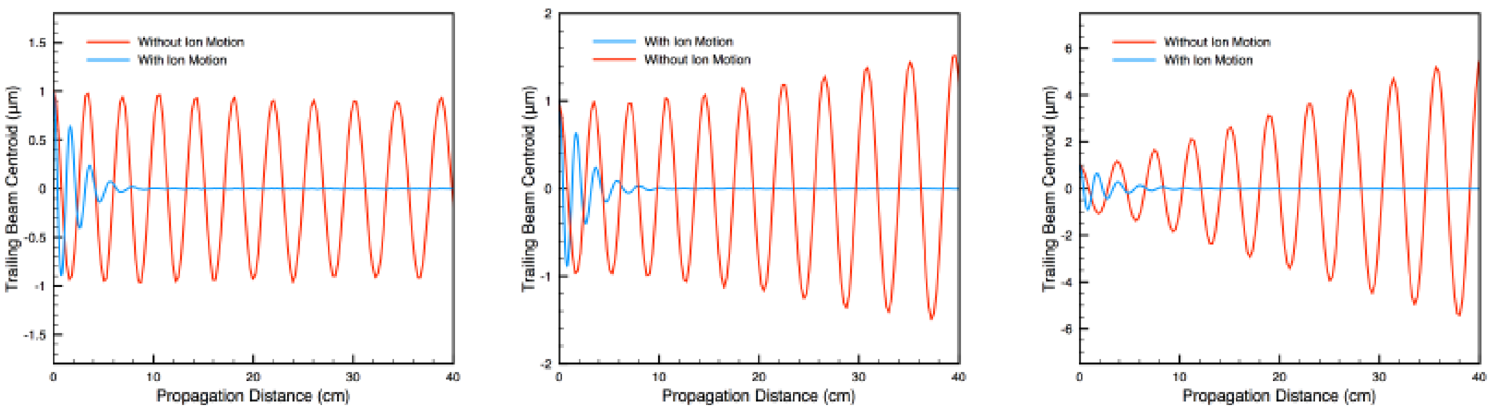
- So far, we considered plasma ions to be stationary (constant transverse focusing).
- In fact, if the bunch density is high enough, the plasma ions are pulled into the electron bunch and create nonlinear focusing.
- Effect was considered first by J. Rosenzweig et al, PRL95, 195002 (2005). Found to be detrimental because of emittance growth.

- However, nonlinear focusing might be helpful to suppress the BBU instability (by allowing some emittance growth)
- Recent simulations performed by Weiming An (UCLA) et al.
  - https://conf.slac.stanford.edu/facet-2-2017/agenda
  - PRL 118, 244801 (2017)

Eliminate the Hosing Instability

UCLA

Head | Center | Tail



$\xi = -\sigma_z$ | $\xi = 0$ | $\xi = \sigma_z$

Drive Beam: E = 10 GeV, I_{peak}=15 kA
σ_r = 0.516 µm, σ_z = 12.77 µm ,
N =1.0 x 10^{10} (1.6 nC), ε_N = 1 µmrad

Trailing Beam: E = 10 GeV, I_{peak}=9 kA
σ_r = 0.516 µm, σ_z = 6.38 µm ,
N =4.33 x 10^9 (0.69 nC), ε_N = 1 µmrad
(transversely offset by 1 µm)

Power efficiency: 50%

Emittance growth: ~a factor of two

See: https://conf.slac.stanford.edu/facet-2-2017/agenda

BNS damping by plasma ions (new idea!)

arXiv:1808.03860

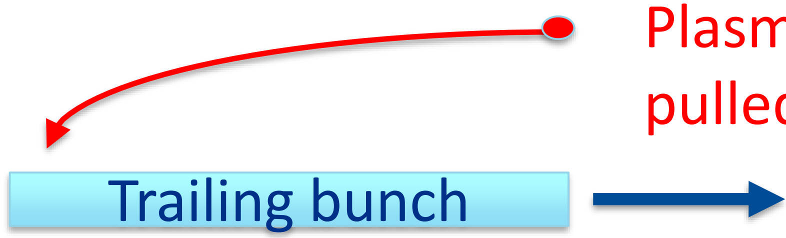
$$\frac{d^2X}{d\mu^2}+\left(1+2\frac{\Delta\omega_\perp}{\omega_\perp}\right)X=\frac{2\eta_t}{L_t^2}\int_0^{\xi}X\left(\xi'\right)\left(\xi-\xi'\right)d\xi'.$$

$$2\frac{\Delta\omega_\perp(\xi)}{\omega_\perp}=\eta_t\frac{\xi^2}{L_t^2}$$

-- the betatron frequency increases from bunch head to tail

This focusing variation is normally achieved by an energy chirp, but in PWA, there may be an additional mechanism – plasma ion mobility

Plasma ions are pulled into the beam



Assuming ion density variation is small:

$$\frac{\Delta n_i(\xi)}{n_i}\approx 2\pi n_t r_i \xi^2 \qquad n_t = N/\ 4\pi L_t\sigma_\perp^2$$

Plasma ions at FACET-II….

- Since $\frac{\Delta\omega_\perp(\xi)}{\omega_\perp} = \frac{\Delta n_i}{2n_i}$ we would like to have $\frac{N r_i L_t}{2\sigma_\perp^2} = \eta_t$

- For FACET-II parameters: 10 GeV, n_0 = 4x10^{16} cm^{-3}
  N = 10^{10} , L = 5 μm

Goal: $\eta_P = 50\%,\ \eta_t \approx 0.13$

For the rms norm emittance 1 μm we should observe BNS damping due to ion moblity (at 50% power efficiency) $\frac{N r_i L_t}{2\sigma_\perp^2} = 0.13$

For the rms norm emittance 10 μm we will not observe BNS damping due to ions (at 50% power efficiency)

$\frac{N r_i L_t}{2\sigma_\perp^2} = 0.013$

These examples are based on hydrogen plasma
arXiv:1808.03860

Conclusions

- We have found a universal efficiency-instability relation for plasma acceleration. Should allow for tolerance and instability analysis without detailed computer simulations.
  - "Efficiency versus instability in plasma accelerators", PRAB 20, 121301 (2017)
  - We considered only ideal "trapezoidal" distributions. Real-life distributions may be worse (from the efficiency perspective).
- In a blowout regime, plasma focusing is just strong enough to keep the instability in check for low power efficiencies (<25%)
  - Even for such efficiencies, external focusing and hollow channels are very challenging because of transverse BBU instability.
  - Presents obvious difficulties for positrons
- Classical BNS damping is possible but external optical systems may limit the momentum spread to ~1% max. Thus, the power efficiency (drive to trailing) can not exceed ~18%.
- BNS damping may be based on ion mobility for some range of bunch and plasma parameters. Can be tested at FACET-II.
  - Preparing an experimental proposal

SBIR funded collaboration with RadiaSoft LLC

"Maximizing the efficiency of plasma-based lepton accelerators"

PI: Stephen Webb, Office of High Energy Physics, Award # DE-SC0018718

Phase 1 period of performance: July 2018 to June 2019

Phase 2 vision: Maximize efficiency of plasma-based accelerators

a) help to understand, quantify and mitigate instabilities

b) computational support for experiments at FACET-II

Phase 1 goals:

- Assess value of impedance, wake functions in plasma
- Demonstrate accurate calculation of impedance via PIC
- Benchmark with theory; help to plan FACET-II exp'ts

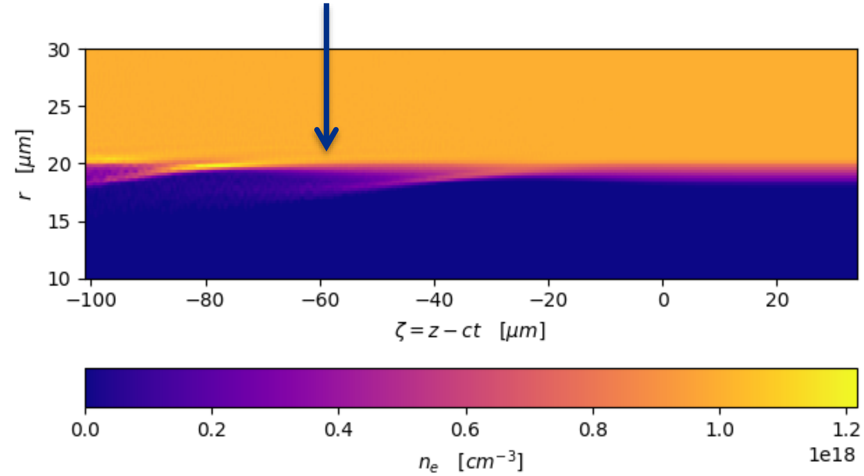
Working with FBPIC

- R. Lehe *et al.*, Comp. Phys. Comm., 203, p. 66 (2016).
- On GitHub, https://github.com/fbpic/fbpic

Initial benchmarking results

- Started with e- beam driver in hollow channel
  - C.A. Lindstrom *et al.*, PRL 120, 124802 (2018).
  - C. Schroeder *et al .*, PRL 82, p. 1177 (1999).
- Quantitative agreement seen for short times

FBPIC simulations of e- beam in hollow channel shows agreement with theory for short times; channel edge evolves, breaking assumptions



Summary

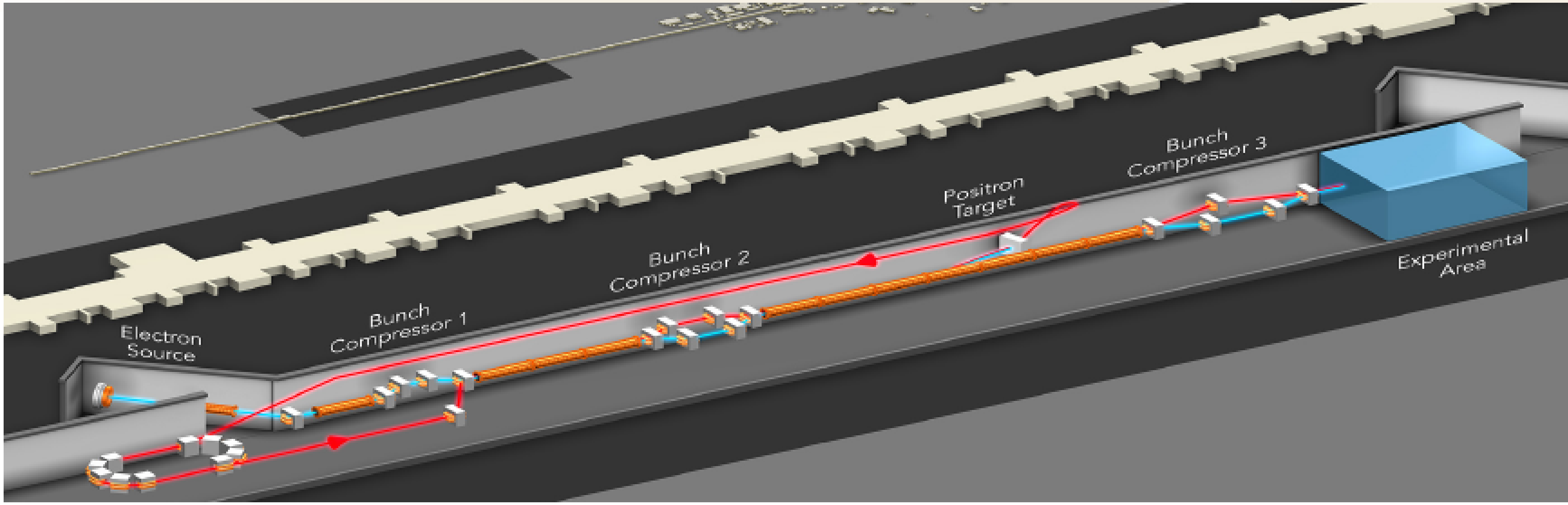
- We now have a very interesting proposal for FACET-II to test our findings of emittance growth vs beam loading, and BNS damping vs beam emittance
  - Large parameter space to explore (small to large beam emittances, ion motion, plasma bubble size, etc)
- Our conclusions require confirmation by computer simulations and by experiments, especially in regimes of smaller bubble sizes.
  - Hence our collaboration with RadiaSoft, SLAC, University of Oslo, CERN and UCLA groups.

Extra slides

FACET-II Technical Design Report SLAC-R-1072

FACET-II Layout and Beams

SLAC



Electron Beam Parameter	*Baseline Design*	*Operational Ranges*
Final Energy [GeV]	*10*	*4.0-13.5*
Charge per pulse [nC]	*2*	*0.7-5*
Repetition Rate [Hz]	*30*	*1-30*
Norm. Emittance $\gamma\varepsilon_{x,y}$ at S19 [µm]	*4.4, 3.2*	*3-6*
Spot Size at IP $\sigma_{x,y}$ [µm]	*18, 12*	*5-20*
Min. Bunch Length σ_z (rms) [µm]	*1.8*	*0.7-20*
Max. Peak current I_{pk} [kA]	*72*	*10-200*

Positron Beam Parameter	*Baseline Design*	*Operational Ranges*
Final Energy [GeV]	*10*	*4.0-13.5*
Charge per pulse [nC]	*1*	*0.7-2*
Repetition Rate [Hz]	*5*	*1-5*
Norm. Emittance $\gamma\varepsilon_{x,y}$ at S19	*10, 10*	*6-20*
Spot Size at IP $\sigma_{x,y}$ [µm]	*16, 16*	*5-20*
Min. Bunch Length σ_z (rms)	*16*	*8*
Max. Peak current I_{pk} [kA]	*6*	*12*

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