Particle Scattering in the Residual Gas

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Typical Residual Gas Estimates at Low Energy

- Lifetime estimate due to single scattering

$$\frac{1}{\tau}=\frac{2\pi r_e^2 c}{\gamma^2\beta^3}\left\langle\left(\frac{\beta_x(s)}{\varepsilon_{mx}}+\frac{\beta_y(s)}{\varepsilon_{my}}\right)\sum_k Z_k\left(Z_k+1\right)n_k(s)\right\rangle_s$$

  - Assumes that the lifetime is dominated by electromagnetic scattering, i.e. the following can be neglected:
    - Nuclear scattering
    - Bremsstrahlung
    - Inelastic scattering on electrons

- Emittance growth due to multiple scattering

$$\frac{d}{dt}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\end{bmatrix}=\frac{2\pi c r_e^2}{\gamma^2\beta^3}\sum_k Z_k\left(Z_k+1\right)\ln\left(\frac{\theta_k^{\max}}{\theta_k^{\min}}\right)\left\langle\begin{bmatrix}\beta_x(s)\\ \beta_y(s)\end{bmatrix}n_k(s)\right\rangle_s$$

where $\theta_k^{\min}=\frac{\hbar}{pa_{atom}}\approx\frac{\sqrt[3]{Z_k}m_e c}{192p}$ is set by atom size

$p=Mc\beta\gamma$ and M is the mass of accelerated particle

$\theta_k^{\max}=\frac{\hbar}{pa_{nucl}}\rightarrow\approx\min\left(\frac{274m_e c}{\sqrt[3]{A_k}\,p},\sqrt{\frac{\varepsilon_{mx,my}}{\beta_{x,y}}}\right)$ is set by nuclear size or the ring acceptance

- For IOTA the top Eq. yields close estimate, but the bottom does not

Electromagnetic Scattering Cross-section

- Classical estimate for screened Coulomb interaction $V(r) = \frac{Ze^2}{r} e^{-r/a}$ yields the scattering angle (small scattering angle approximation):

$$\theta_\perp \approx \frac{4Zr_p}{\gamma\beta^2 r} \exp\left(-\frac{(r/a)^{1.5}}{0.633\sqrt{9+(r/a)^{1.6}}} \right)$$

  - Although the scattering angle decays exponentially $\sigma_{tot}(r)$ diverges

- Quantum mechanical calculation in the Born approximation yields:

$$\frac{d\sigma}{d\Omega} = \frac{4Z^2 r_p^{\,2}}{\gamma^2\beta^4\left(\theta_\perp^{\,2} + \theta_m^{\,2}\right)^2}$$

  - Adding scattering on electrons we obtain

$$\frac{d\sigma}{d\Omega} = \frac{4Z(Z+1) r_p^{\,2}}{\gamma^2\beta^4\left(\theta_x^{\,2} + \theta_y^{\,2} + \theta_m^{\,2}\right)^2}$$

$$\boxed{\frac{d\sigma}{d\theta_x} = \frac{2\pi Z(Z+1) r_p^{\,2}}{\gamma^2\beta^4\left(\theta_x^{\,2} + \theta_m^{\,2}\right)^{3/2}}} \qquad \boxed{\sigma_{tot} = \frac{4\pi Z(Z+1) r_p^{\,2}}{\gamma^2\beta^4\theta_m^{\,2}}} \qquad \boxed{\theta_m \equiv \theta_k^{\min}}$$

  - Total cross-section is finite
  - Typically, the maximum scattering $\theta_k^{\max}$ is larger than the acceptance and can be neglected

Common Treatment of Single and Multiple Scattering

- Separation of single and multiple scattering simplifies the matter but is not adequate in many applications
  - In particular, it does not allow to describe accurately non-Gaussian tails near the core
- Integrodifferential equation addresses this problem

$$\frac{\partial f}{\partial t}-\lambda\frac{\partial(If)}{\partial I}=\beta c\sum_k\left\langle\int_{-\infty}^{\infty}\left(\left.\frac{d\sigma^k}{d\theta_x}\right|_{(\theta-\theta')}-\sigma_{tot}^k\delta(\theta-\theta')\right)n_k(s)f\,\delta(x-x')d\theta' dx'\right\rangle_{\varphi,s}$$

  - where averaging is performed over betatron motion and ring circumference
  - the phase and the action are determined as: $I=\frac{\beta_x\theta_x^{\,2}}{2},\quad x=\sqrt{2I\beta_x}\cos\varphi,\quad \theta_x=\sqrt{\frac{2I}{\beta_x}}\sin\varphi.$
- In further consideration we assume
  - One gas species with uniform distribution over ring
  - Smooth lattice approximation: $\beta_x(s)=\beta_x$

$$\frac{\partial f}{\partial t}-\lambda\frac{\partial(If)}{\partial I}=\frac{2\pi Z(Z+1)r_p^{\,2}nc}{\gamma^2\beta^3}\left\langle\int_{-\infty}^{\infty}\left(\frac{1}{\left(\left(\theta_x-\theta_x'\right)+\theta_{\mathrm{m}}^2\right)^{3/2}}-\frac{2}{\theta_m^{\,2}}\delta(\theta-\theta')\right)f\,\delta(x-x')d\theta' dx'\right\rangle_{\varphi}.$$

Kernel of the Integrodifferential Equation

- Transition to the action phase variables and integration over φ' yields

$$\frac{\partial f}{\partial t}-\lambda\frac{\partial(If)}{\partial I}=B\int_0^\infty\left(\frac{1}{2\pi}\int_0^\pi\frac{d\varphi}{\left(I'+I-2\sqrt{II'}\cos\varphi+I_m\right)^{3/2}\sqrt{I'+I-2\sqrt{II'}\cos\varphi}}-\frac{1}{I_m}\delta(I'-I)\right)f'dI'$$

$$B=\frac{2\pi Z(Z+1)r_p^{\,2}nc}{\gamma^2\beta^3}\beta_x\,,\quad I_m=\frac{\beta_x\theta_m^{\,2}}{2}\,,\quad f(I,t)\equiv f\,,\quad f(I',t)\equiv f'$$

As we'll see: D=B ln(..)

- Rewrite the equation in the form

$$\frac{\partial f}{\partial t}=\lambda\frac{\partial}{\partial I}(If)+B\left(\int_0^{I_b}W(I,I')f(I',t)\mathrm{d}I'-\frac{f}{I_m}\right),\quad I<I_b$$

where the kernel is

$$W(I,I')=\frac{1}{2\pi}\int_0^\pi\frac{d\varphi}{\left(I'+I-2\sqrt{II'}\cos\varphi+I_m\right)^{3/2}\sqrt{I'+I-2\sqrt{II'}\cos\varphi}}$$

- The kernel can be expressed through the elliptic integrals

$$W_I(I,I')=\frac{\hat{W}\left(I/I_m,I'/I_m\right)}{I_m^{\,2}},\quad \hat{W}(x,x')=\frac{1}{\pi\left(\sqrt{x}+\sqrt{x'}\right)\sqrt{\left(\sqrt{x}-\sqrt{x'}\right)^2+1}}\left(K\left(k^2\right)-\frac{\left(\sqrt{x}+\sqrt{x'}\right)^2}{\left(\sqrt{x}+\sqrt{x'}\right)^2+1}E\left(k^2\right)\right),\ k^2=\frac{4\sqrt{xx'}}{\left(\sqrt{x}+\sqrt{x'}\right)^2\left(\left(\sqrt{x}-\sqrt{x'}\right)^2+1\right)}$$

Properties of the Kernel

- The kernel is symmetric:

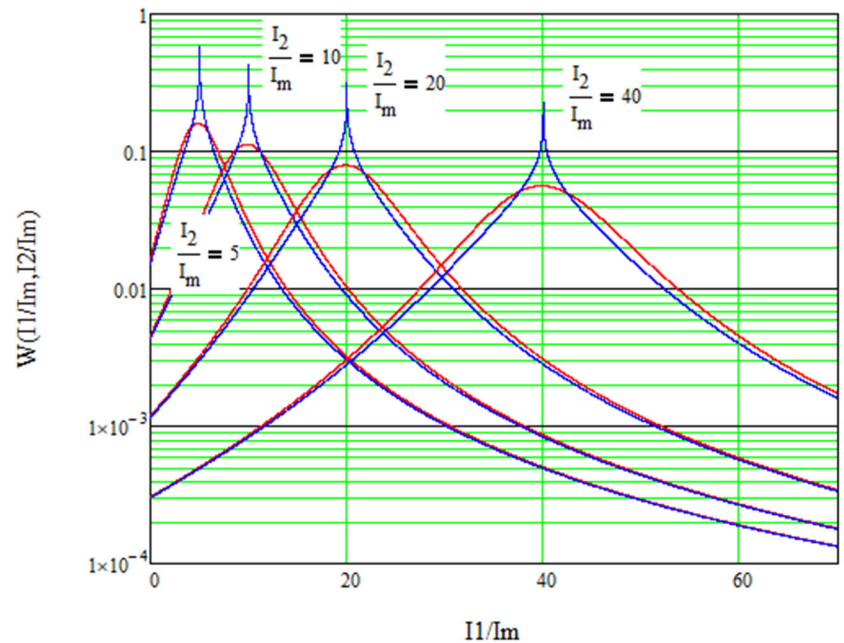
$$W(I,I') = W(I',I)$$

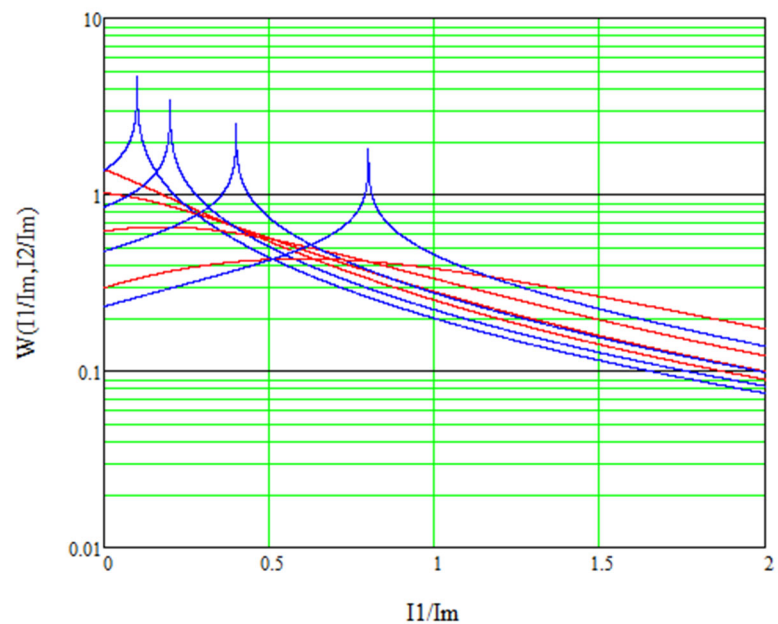
- The kernel conserves the particle number: $\int_0^\infty W(I,I') f(I',t) \mathrm{d}I' = \frac{1}{I_m}$
- The kernel logarithmically diverges at $I \to I'$
- At Tevatron times for numerical calculations we used

$$W_I(I,I') = \frac{1}{2} \frac{I + I' + 1/2}{\left((I - I')^2 + (I + I') I_m + I_m{}^2 / 4 \right)^{3/2}}$$

This approximation

- conserves the number of particles
- Has correct asymptotic in the tails
- works well if the distribution width is much larger than I_m ($I, I' \gg I_m$)

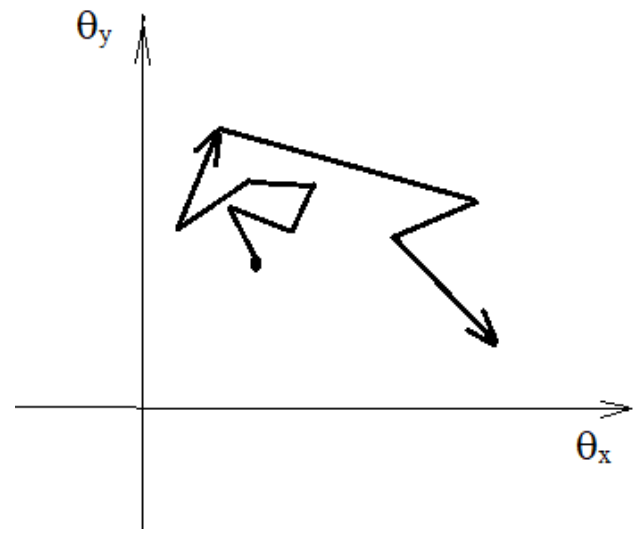


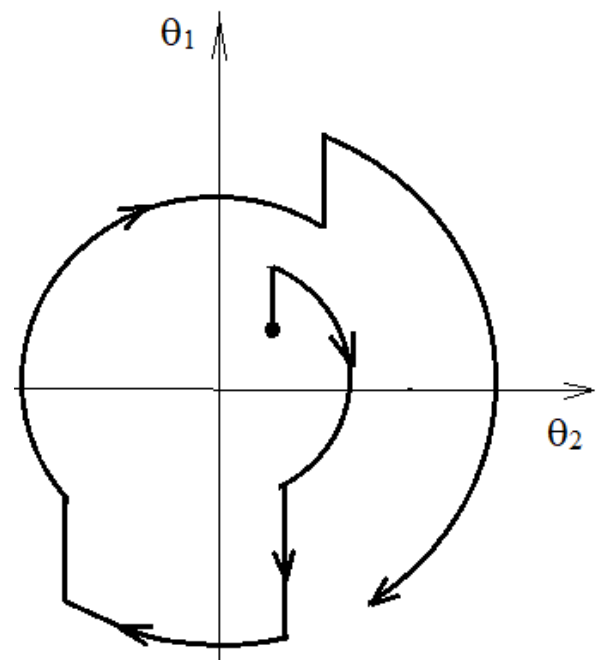


Scattering in Medium vs Scattering in Phase Space

- From math point of view the particle scattering in the plane of transverse angles and the particle scattering in the phase space of an accelerator (one plane scattering) are identical
  - The only difference are the cross-sections

$$\frac{d\sigma}{d\Omega} d\theta_x d\theta_y \rightarrow \frac{1}{\sqrt{\theta_1^2 + \theta_2^2}} \frac{d\sigma}{d\theta_x} d\theta_1 d\theta_2$$





Gas Scattering in the Absence of Cooling

Particle Scattering in the Absence of Cooling

- In the absence of cooling the integrodifferential equation has analytical solution
- The solution uses the same idea as for the Moliere scattering
  - Rewrite integrodifferential equation in θ_1 & θ_2 variables instead of action ($\theta_1 = \sqrt{I}\cos\varphi, \theta_2 = \sqrt{I}\sin\varphi$)

$$\frac{\partial f(\theta_1,\theta_2)}{\partial t} = \frac{2B}{\beta_x}\left(\int_{-\infty}^{\infty}\frac{f(\theta_1',\theta_2')}{\left((\theta_1-\theta_1')^2+(\theta_2-\theta_2')^2+\theta_m{}^2\right)^{3/2}}\frac{d\theta_1' d\theta_2'}{2\pi\sqrt{(\theta_1-\theta_1')^2+(\theta_2-\theta_2')^2}}-\frac{f(\theta_1,\theta_2)}{\theta_m{}^2}\right)$$

  - Perform 2D Fourier transform of equation and initial distribution
  - Perform integrations which result in a dependence of harmonics on ***t***

$$f_{k_1,k_2}(t) = f_{k_1,k_2}(0)\exp\left(\frac{Bt}{I_m}\left(\pi k\theta_m\left(I_0\left(\frac{k\theta_m}{2}\right)K_1\left(\frac{k\theta_m}{2}\right)-I_1\left(\frac{k\theta_m}{2}\right)K_0\left(\frac{k\theta_m}{2}\right)\right)-1\right)\right)$$

  - Perform inverse Fourier transform
  - Account for axial symmetry and integrate over angle

$$\boxed{f(\theta,t) = 2\pi\int_0^{\infty} f_k(0)\exp\left(\frac{Bt}{I_m}\left(\frac{k\theta_m}{2}\left(I_0\left(\frac{k\theta_m}{2}\right)K_1\left(\frac{k\theta_m}{2}\right)-I_1\left(\frac{k\theta_m}{2}\right)K_0\left(\frac{k\theta_m}{2}\right)\right)-1\right)\right)J_0(k\theta)kdk}$$

Scattering for Point-like Initial Distribution

- Initial distribution $f(\theta_1,\theta_2)_{t=0} = \delta(\theta_1)\delta(\theta_1) \Rightarrow f_{k_x,k_y}\Big|_{t=0} = \dfrac{1}{4\pi^2}$
- Introducing dimensionless time, and renormalizing scattering angle

$$\boxed{\tau = \frac{Bt}{I_m} \equiv N_{collisions} = n\sigma_{tot} \mathrm{v} t\,, \quad \Theta = \frac{\theta}{\theta_m}}$$

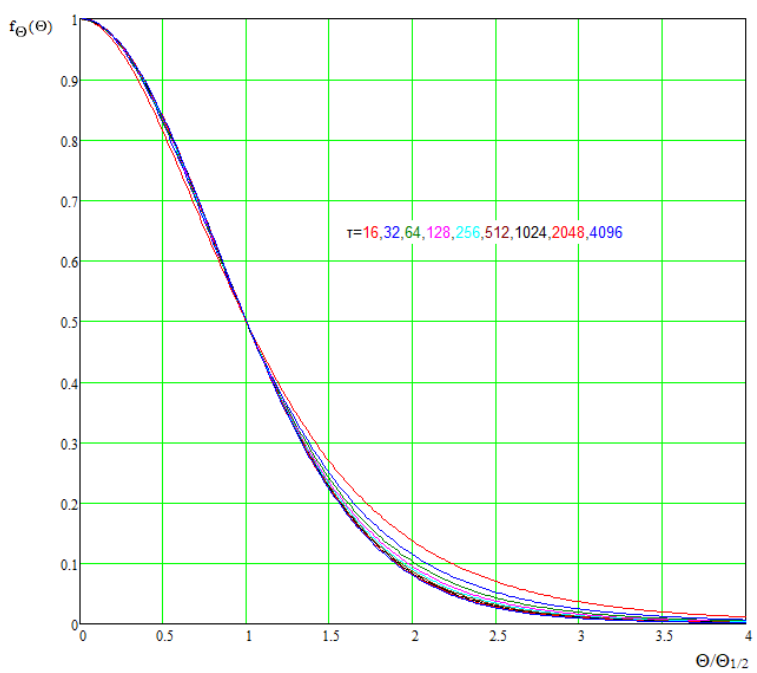
$$\Rightarrow \quad \boxed{f_\Theta(\Theta,\tau) = \frac{1}{2\pi}\int_0^\infty J_0(\Theta x)\exp\left(\left(\frac{x}{2}\left(I_0\left(\frac{x}{2}\right)K_1\left(\frac{x}{2}\right) - I_1\left(\frac{x}{2}\right)K_0\left(\frac{x}{2}\right)\right) - 1\right)\tau\right)x\,dx}$$

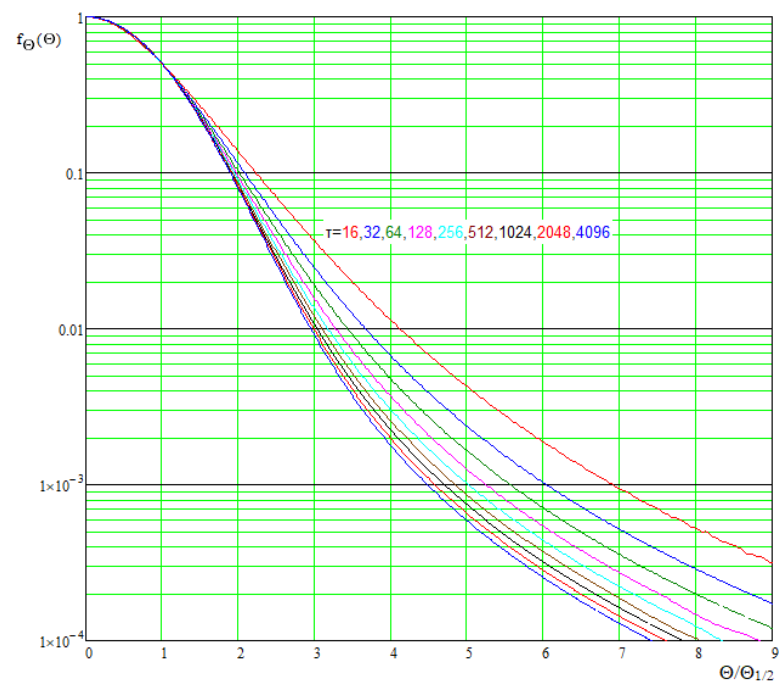
- Distribution normalization

$$2\pi\int_0^\infty f(\Theta)\Theta\, d\Theta = 1$$

- Relationship with action

$$\boxed{I = I_m \Theta^2}$$



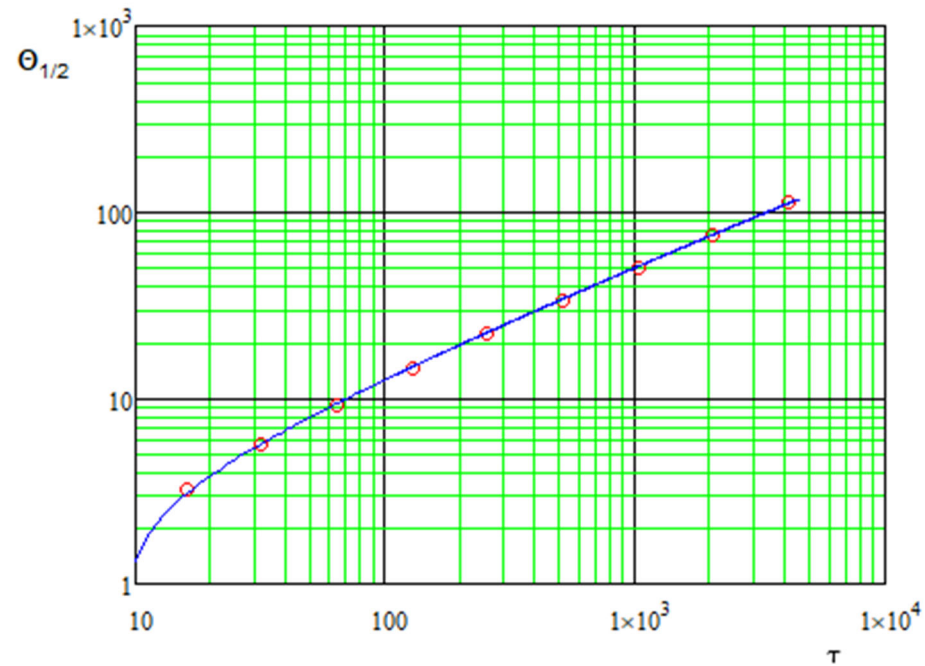


Scattering for Point-like Initial Distribution (2)

- Rare strong kicks result in
  - Central limit theorem does not work
  - Long non-Gaussian tails
- Logarithmic dependence of distribution width on time

$$\Theta_{1/2} = \sqrt{0.85\tau \ln^{0.7}(\tau/9)}, \quad \tau \geq 16.$$

$$\Rightarrow \theta_{1/2} \equiv \sqrt{\frac{2Dt}{\beta_x}}, \quad D = 0.85B\ln^{0.7}(Bt/9I_m)$$



- Compare to

PDG particle data group

If we define

$$\theta_0 = \theta^{\text{rms}}_{\text{plane}} = \frac{1}{\sqrt{2}}\theta^{\text{rms}}_{\text{space}} . \qquad (27.11)$$

then it is sufficient for many applications to use a Gaussian approximation for the central 98% of the projected angular distribution, with a width given by [32,33]

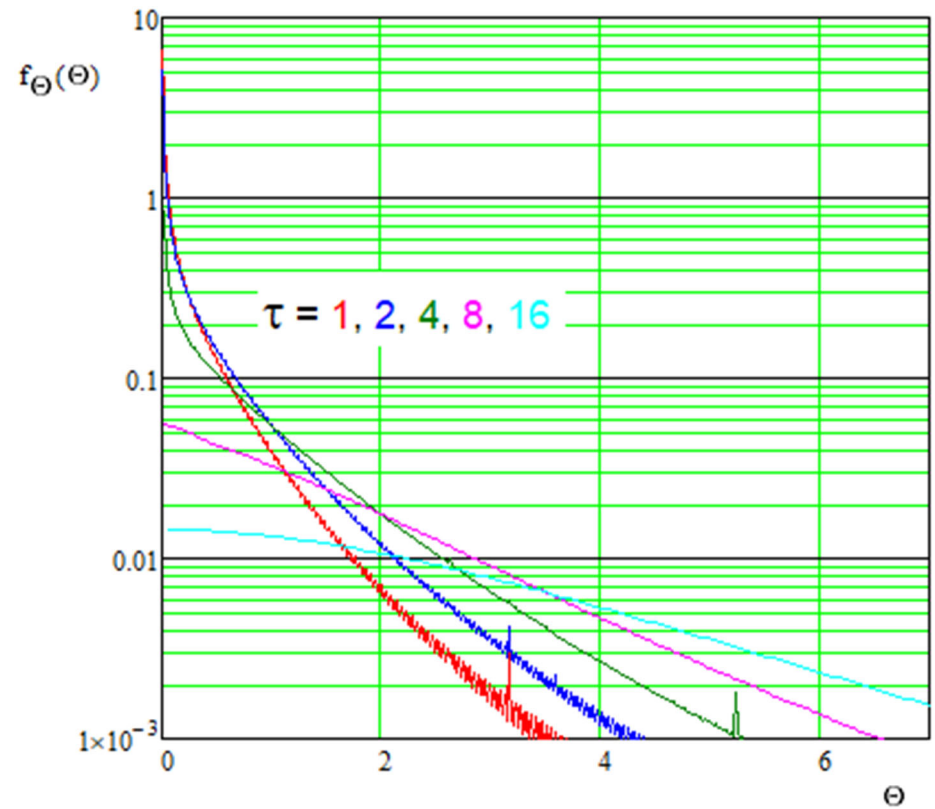
$$\theta_0 = \frac{13.6\ \text{MeV}}{\beta cp}\, z\, \sqrt{x/X_0}\left[1 + 0.038\ln(x/X_0)\right] . \qquad (27.12)$$

Scattering for Point-like Initial Distribution: Small τ

- Integral has poor convergence for small $\tau \equiv N_{collisions}$ because there is a δ-function left from the initial distribution
  - Regularization helps for convergence

$$f_\Theta(\Theta,\tau) = \delta(\Theta)e^{-\tau} + \frac{1}{2\pi}\int_0^\infty J_0(\Theta x)\left[\exp\left(\left(\frac{x}{2}\left(I_0\left(\frac{x}{2}\right)K_1\left(\frac{x}{2}\right) - I_1\left(\frac{x}{2}\right)K_0\left(\frac{x}{2}\right)\right) - 1\right)\tau\right) - e^{-\tau}\right]x\,dx$$

- The shape of the distribution experiences fundamental changes for $\tau < 16$
  - from the Gaussian with tails to the distribution diverging at $\Theta \to 0$
- Therefore there is no straightforward way to determine the distribution width for $\tau < \sim 16$



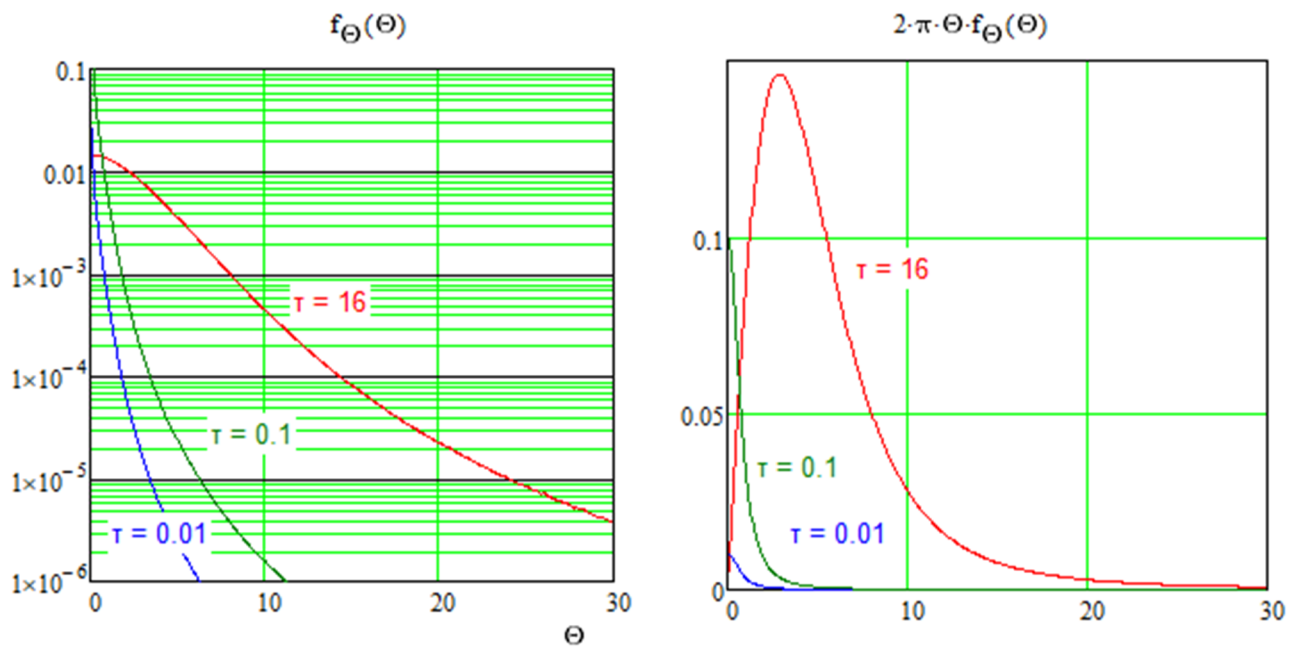
Scattering for Point-like Initial Distribution: Small τ (2)

- For very small time (τ<<1) we can neglect secondary scatterings

$$f_\Theta(\Theta,\tau) \simeq \frac{1}{2\pi\Theta}\left(\frac{\tau}{\left(1+\Theta^2\right)^{3/2}} + e^{-\tau}\delta(\Theta)\right), \quad \tau \ll 1$$

Remind: the distribution normalization is: $2\pi\int_0^\infty f(\Theta)\Theta d\Theta = 1$

- This equation makes reasonably good approximation for τ < 0.5



- Rms scattering angle diverges logarithmically: $\left\langle\Theta^2\right\rangle = 2\pi\int_0^{\Theta_{max}} f(\Theta)\Theta^3 d\Theta \propto \ln\Theta_{max}$

Gas Scattering in the Presence of Cooling

Gas Scattering in the Presence of Cooling

$$\frac{\partial f}{\partial t} = \lambda \frac{\partial}{\partial I}(If) + B\left(\int_0^{I_b} W(I,I')f(I',t)\mathrm{d}I' - \frac{f}{I_m}\right), \quad I < I_b$$

- Transiting to the dimensionless action $\hat{I} = I / I_m$ and time $\tau = \lambda t$ and looking for the equilibrium distribution one obtains:

$$\frac{d}{d\hat{I}}\left(\hat{I}f\right) + \hat{B}\left(\int_0^{\hat{I}_b} \hat{W}(\hat{I},\hat{I}')f'\,\mathrm{d}\hat{I}' - f\right) = 0$$

- Regrouping we have: $\left(\hat{B}-1\right)f - \hat{I}\frac{df}{d\hat{I}} = \hat{B}\int_0^{\hat{I}_b} \hat{W}(\hat{I},\hat{I}')f'\,\mathrm{d}\hat{I}'$

  where $\hat{B} = \beta c n \sigma_{tot} / \lambda$ is the number of collisions in one damping time

  - The RHS is always positive => if $\hat{B} < 1$ then $\partial f / \partial \hat{I}$ should by negative and approach $-\infty$ for $\hat{I} \to 0$
  - This condition separates to classes of solutions
    - Finite for $\hat{B} > 1$
    - Diverging at $\hat{I} \to 0$ for $\hat{B} < 1$

Equilibrium Distribution for Very Strong Cooling

- In the case of strong damping, $\hat{B} \ll 1$, the distribution function can be obtained if we assume that all scattering happens from zero amplitude
- Substituting $f' \ll \delta(\hat{I}')$ & $\hat{W}(\hat{I},0) = \dfrac{1}{2\left(\hat{I}+1\right)^{3/2}\sqrt{\hat{I}}}$ into $\dfrac{\partial}{\partial \hat{I}}\left(\hat{I}f\right) + \hat{B}\left(\int_0^{\hat{I}_b} \hat{W}(\hat{I},\hat{I}')f'\,\mathrm{d}\hat{I}' - f\right) = 0$

  and integrating one obtains:

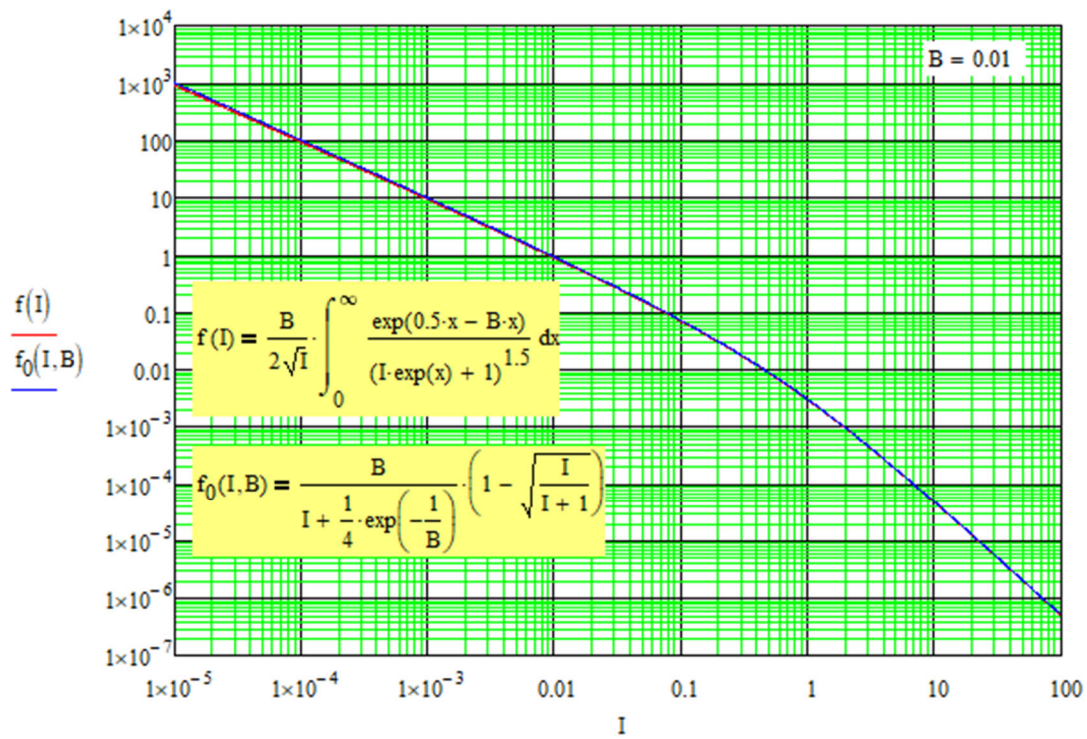
$$\boxed{f(\hat{I}) \approx \frac{\hat{B}}{\hat{I}}\left(1-\sqrt{\frac{\hat{I}}{\hat{I}+1}}\right) \xrightarrow[\text{and to normilize to 1}]{\text{Correct to avoid divergence}} \frac{\hat{B}}{\hat{I}+e^{-1/\hat{B}}/4}\left(1-\sqrt{\frac{\hat{I}}{\hat{I}+1}}\right)}$$

- Rms action is

$$\boxed{\bar{I} \approx \frac{\hat{B}}{2}\ln\left(\frac{4\hat{I}_b}{e}\right), \quad \hat{I}_b \gg 1, \quad \hat{B} \ll 1.}$$

- More sophisticated solution is

$$\boxed{f(\hat{I}) = \frac{\hat{B}}{2\sqrt{\hat{I}}}\int_0^{\infty} \frac{e^{x/2}e^{-\hat{B}x}}{\left(\hat{I}e^{x}+1\right)^{3/2}}dx}$$

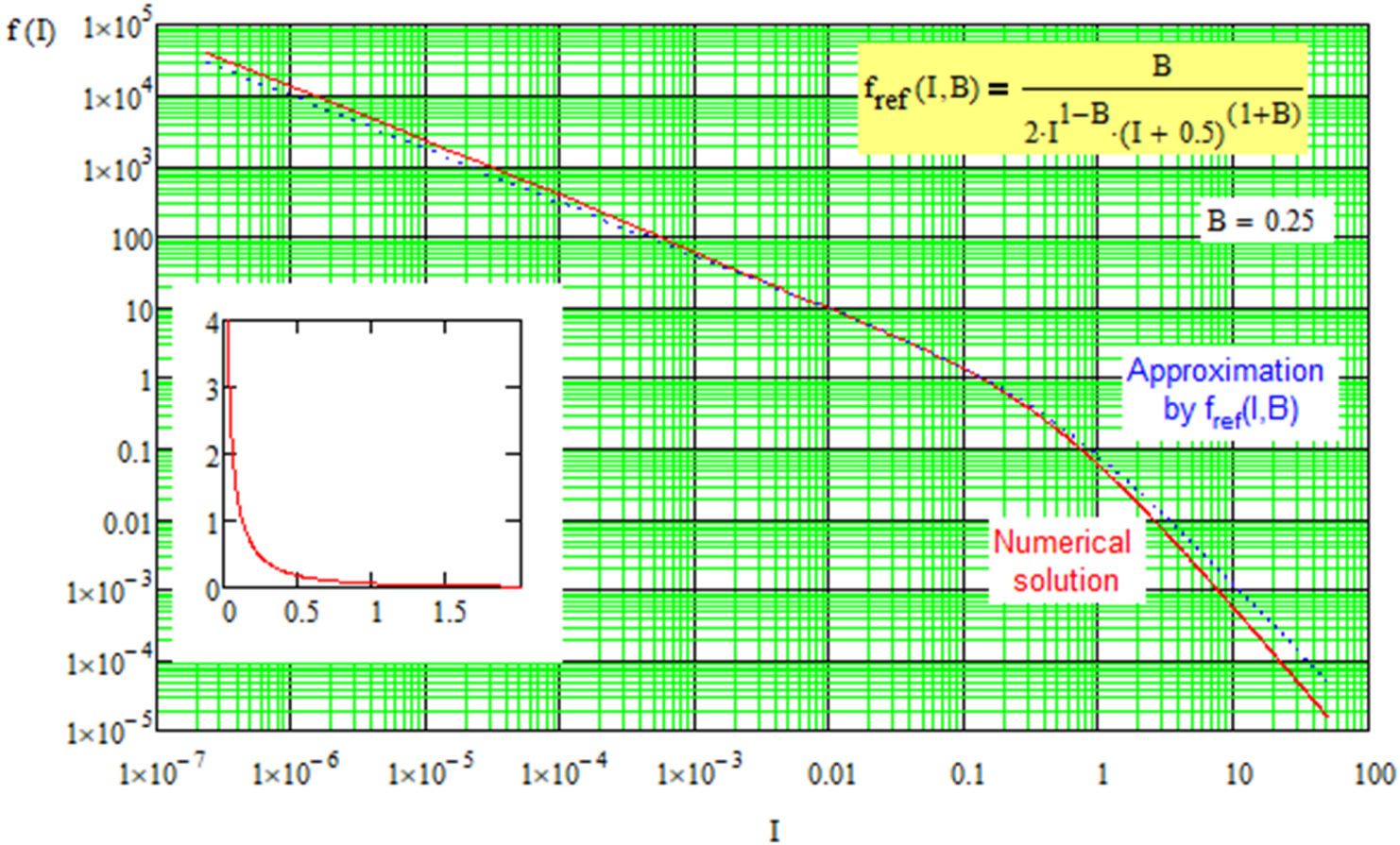


Equilibrium Distribution for Strong Cooling

- For $\hat{B} < 1$ we can roughly approximate the solution by the following equation

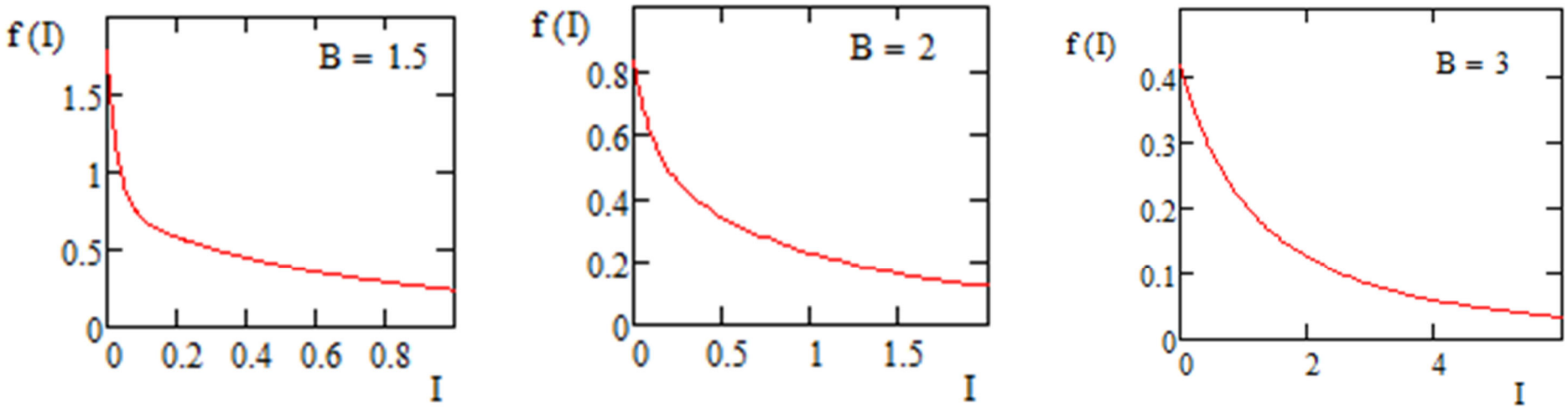
$$f(\hat{I}) \approx \frac{\hat{B}}{2\hat{I}^{1-\hat{B}}\left(\hat{I}+1/2\right)^{1+\hat{B}}}, \quad 0 < \hat{B} < 1, \quad \int_0^\infty f(\hat{I})d\hat{I} = 1$$

- This approximation coincides well with the previous slide equation for $\hat{B} < 0.1$



Equilibrium Distribution for Weak Cooling

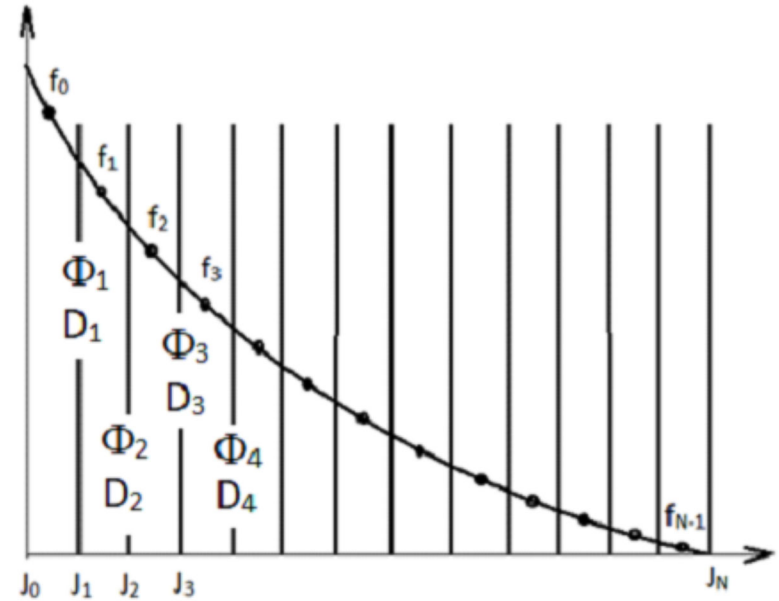
- Divergence at zero action disappears for $\hat{B} > 1$



- For $\hat{B} > 3$ the distribution becomes more like the Gaussian with non-Gaussian tail which value is decreasing with $\hat{B}$ increase

Numerical Solution

Numerical Solution



- Split action into the boxes;
- Find transition probabilities between boxes

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial \hat{I}}\left(\hat{I} f\right) + \hat{B}\left(\int_0^{I_b} W(\hat{I},\hat{I}') f(\hat{I}',t)\mathrm{d}\hat{I}' - f\right), \quad I < I_b$$

$$\Rightarrow \frac{\partial f_n}{\partial \tau} = \frac{f_{n+1} - f_{n-1}}{2\Delta J} + \hat{B}\sum_{m=0}^{N-1} w_{nm} f_m$$

- Reduce the difference scheme to matrix multiplication

$$\Rightarrow \boxed{\mathbf{f}_{k+1} = \mathbf{f}_{k+1} + \left(\lambda \mathbf{\Lambda} + \hat{B}\mathbf{w}\right)\mathbf{f}_k \Delta\tau}$$

- For far away cells

$$w_{nm} = \frac{1}{\Delta I}\int_{n\Delta I}^{(n+1)\Delta I} d\hat{I} \int_{m\Delta I}^{(m+1)\Delta I} \hat{W}(\hat{I}',\hat{I})\mathrm{d}\hat{I}' \approx \hat{W}\left(\left(n+\frac{1}{2}\right)\Delta J, \left(m+\frac{1}{2}\right)\Delta J\right)\Delta J, \quad n \neq m, \quad n \neq m \pm 1.$$

- For nearby cells we assume the distribution function changing linearly =>

$$w_{n,n+1} = \Delta I\left(\int_0^1 W_a(x)x^2 dx + \int_1^2 W_a(x)x(2-x)dx\right),$$

$$W_a(x) = \frac{1}{2\pi\sqrt{\Delta J(n+1)}}\frac{1}{\sqrt{f(x,n)}}\left(K\left(\frac{1}{f(x,n)}\right) - \frac{4(n+1)\Delta J}{1+4(n+1)\Delta J}E\left(\frac{1}{f(x,n)}\right)\right), \quad f(x,n) = 1 + \frac{\Delta J x^2}{4(n+1)}$$

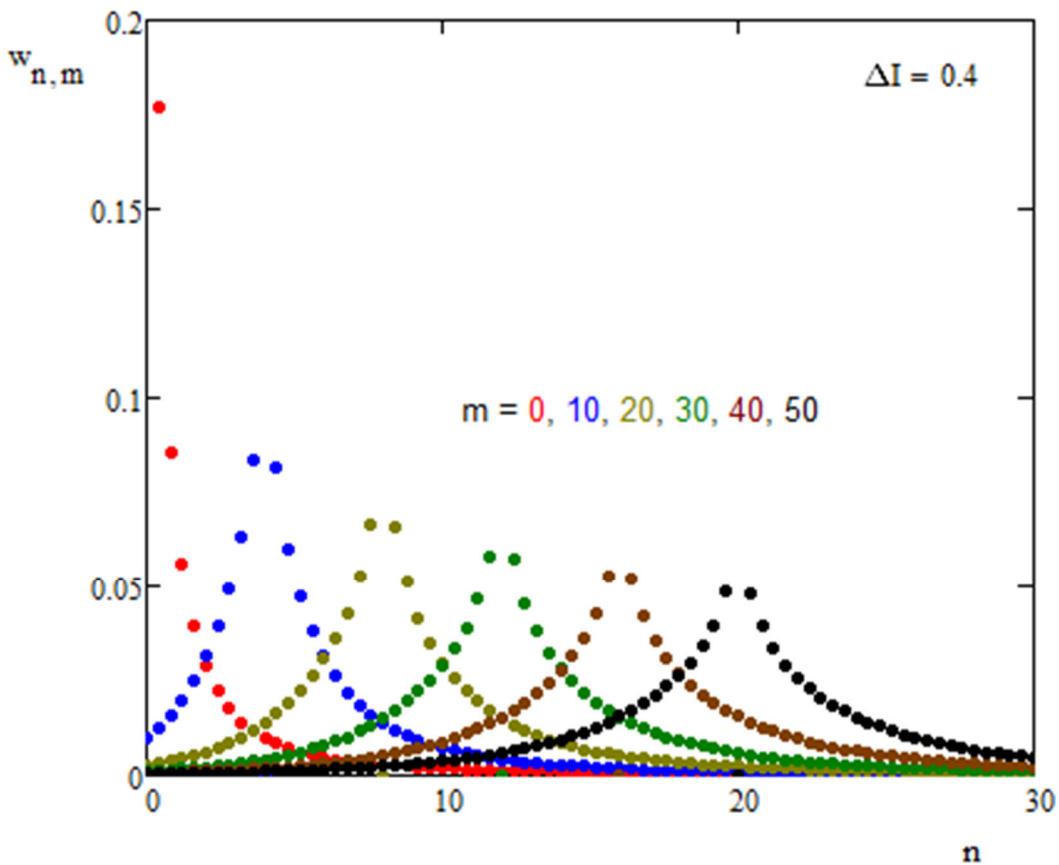
Numerical Solution (2)

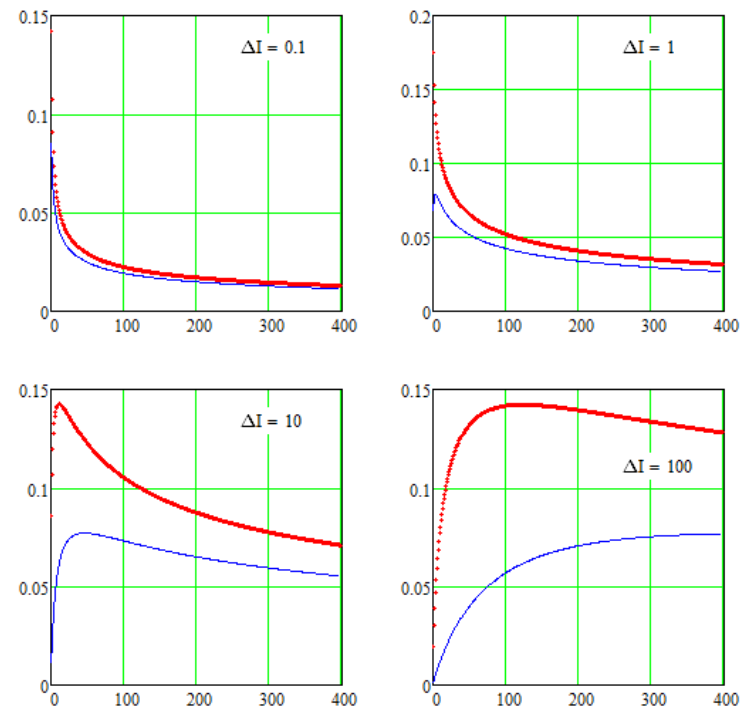
- Other elements can be found using: $w_{n,n+1} = w_{n+1,n}$, $w_{n,n} = -\sum_{m \neq n} w_{n,m}$

  The 2nd equation assumes particle conservation.

  If required it is straightforward to account for the particle loss

$$w_{n,n} = -\sum_{m \neq n} w_{n,m} - \Delta J \sum_{m=N}^{\infty} \hat{W}\left(\left(n+\frac{1}{2}\right)\Delta J, \left(m+\frac{1}{2}\right)\Delta J\right)$$





Dependence of $w_{n,n+1}$ (red points) and $w_{n,n+2}$ (blue points) on n for different ΔI; n (which counts matrix elements) starts from 0.

Practical Applications

Vacuum and Noise of Dipoles in Tevatron Run II

- To built the luminosity evolution model we needed to know all sources of beam diffusion:
  - IBS,
  - RF noise,
  - scattering at the residual gas
  - noise in dipoles ($\Delta B/B \sim 10^{-9}$ - 10^{-10} is a big deal)
- Common treatment of single and multiple scattering was developed to understand a contribution of magnetic noise in dipoles
  - This noise generates Gaussian distribution while scattering generates non-Gaussian tails
- The measurements we done with small intensity continuous beam to avoid IBS
- Only measurements at injection energy could be done because of quenching
- The conclusion was: at least 80% of the emittance growth at 150 MeV comes from the gas scattering for small beam current (no IBS)

Vacuum and Noise of Dipoles in Tevatron Run II (2)

- First, we scraped the beam to ~75% to create step in the distribution
- Waited ~30-60 minutes to get the diffusion to smear the distribution
- Final entire beam scraping yielded the integral distribution
- Comparison with theory exhibited non-Gaussian tails which value proved that at least 80% of emittance growth is related to the gas scattering

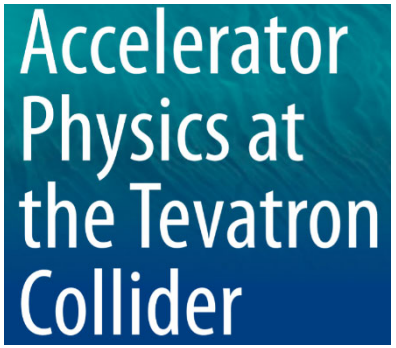
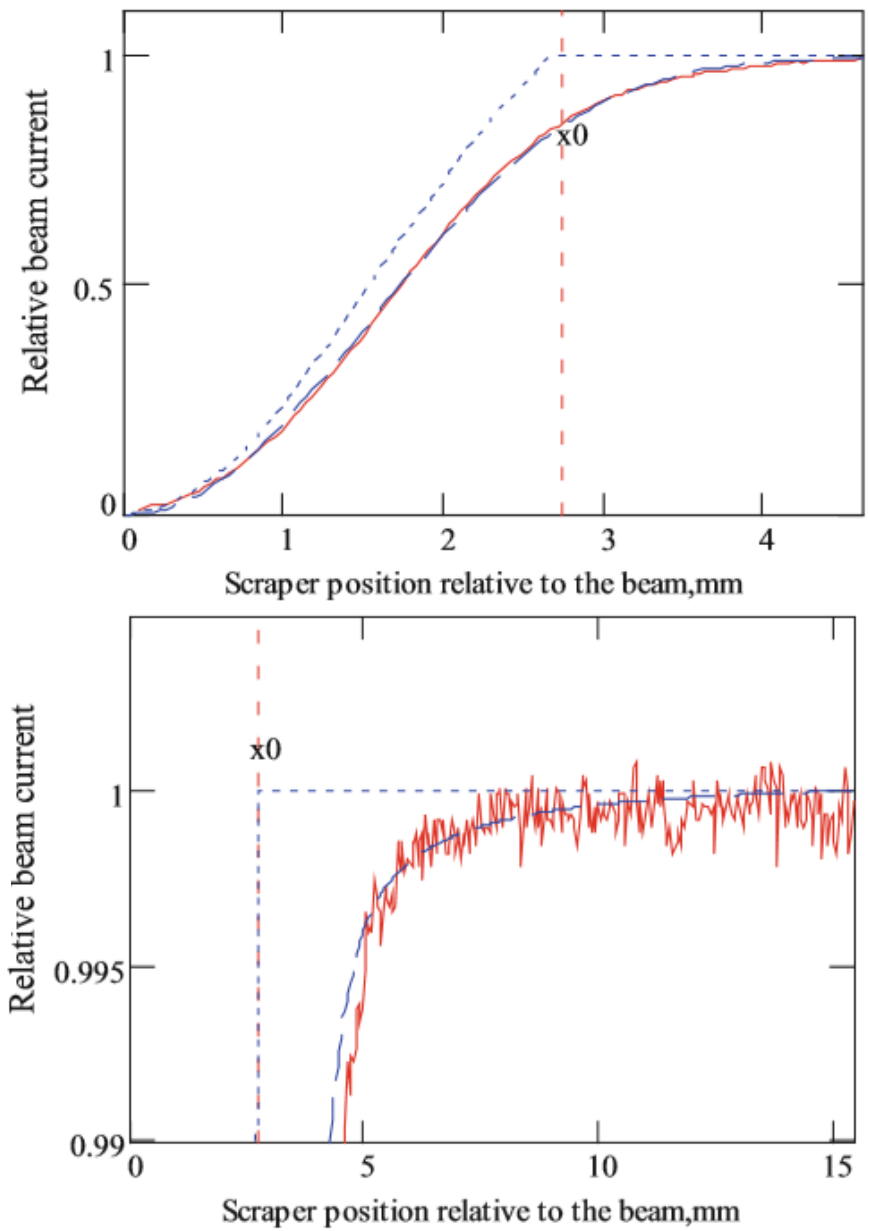


Fig. 6.9 Dependence of the beam current on the vertical scraper position for the beam core (*top*) and beam tails (*bottom*); *solid line*—measurements, *dashed line*—computer simulations for $L_c = 8.6$, *dotted line*—the dependence which would be measured with the initial distribution; x_0 marks the final scraper position at the initial scraping



Vacuum and Noise of Dipoles in Tevatron Run II (3)

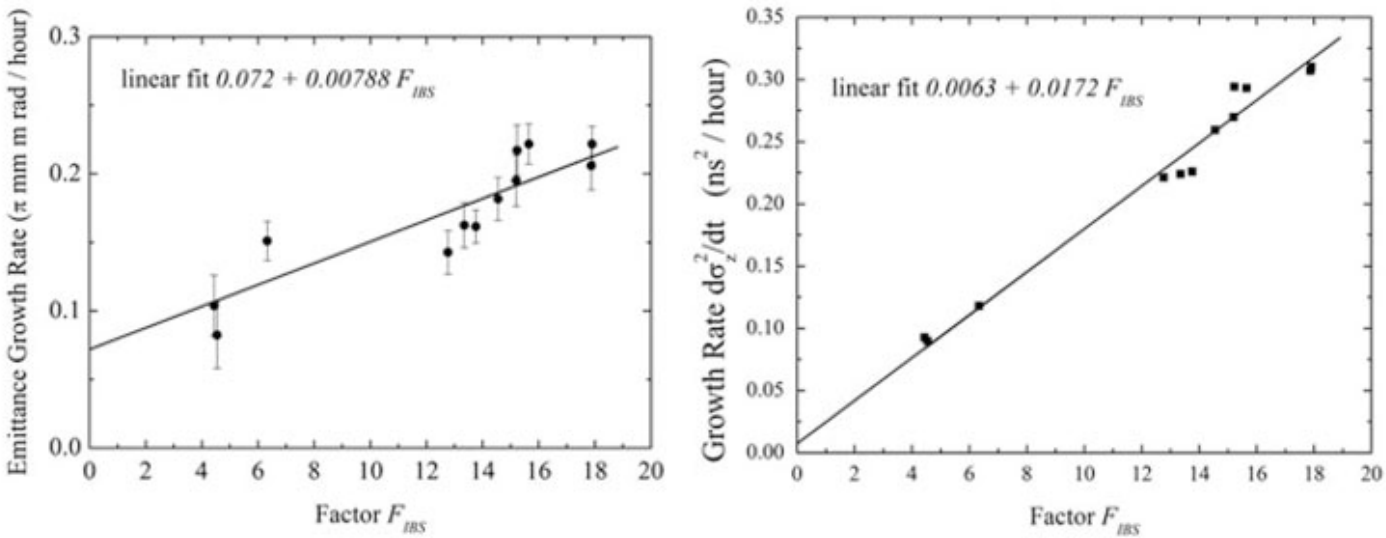
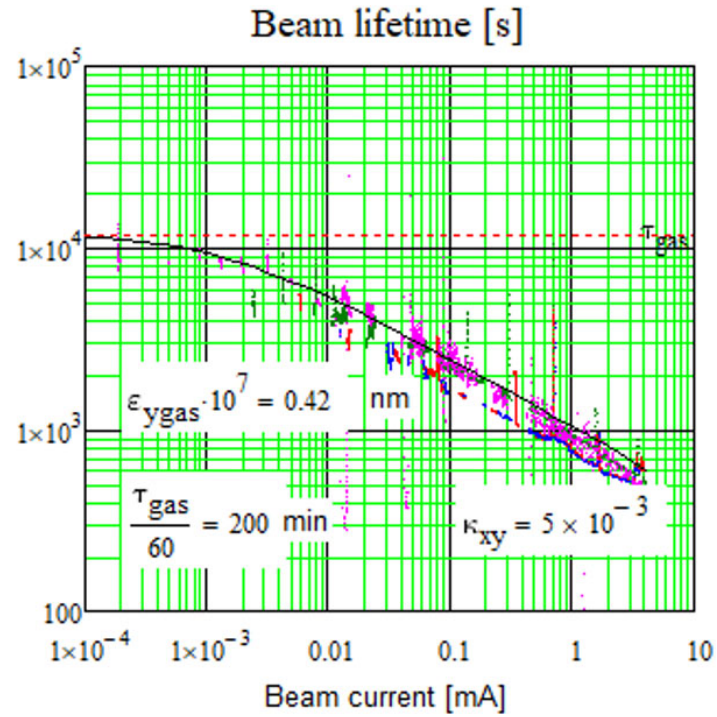


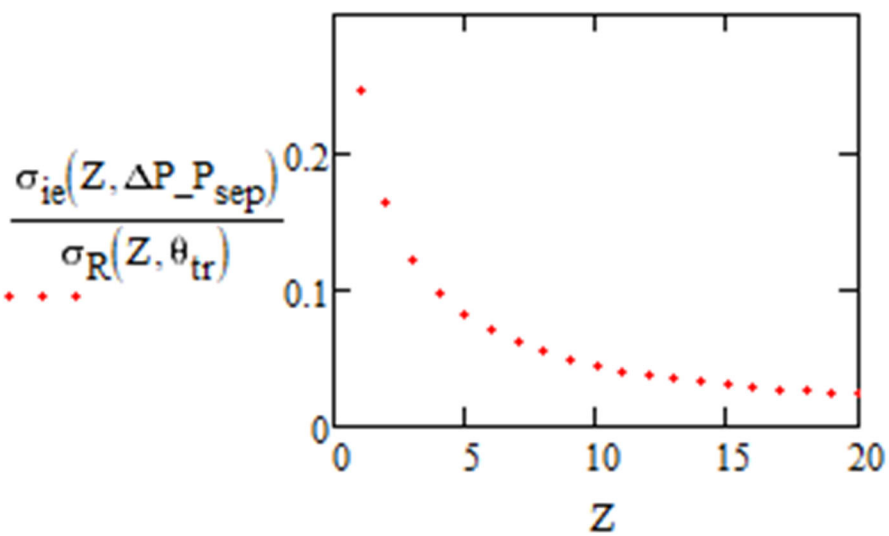
Fig. 6.12 Vertical emittance growth rates (rms, norm.) of proton bunches vs the IBS factor F_{IBS} (*left*); the rms bunch length growth rates vs the IBS factor F_{IBS} (*right*) [20]

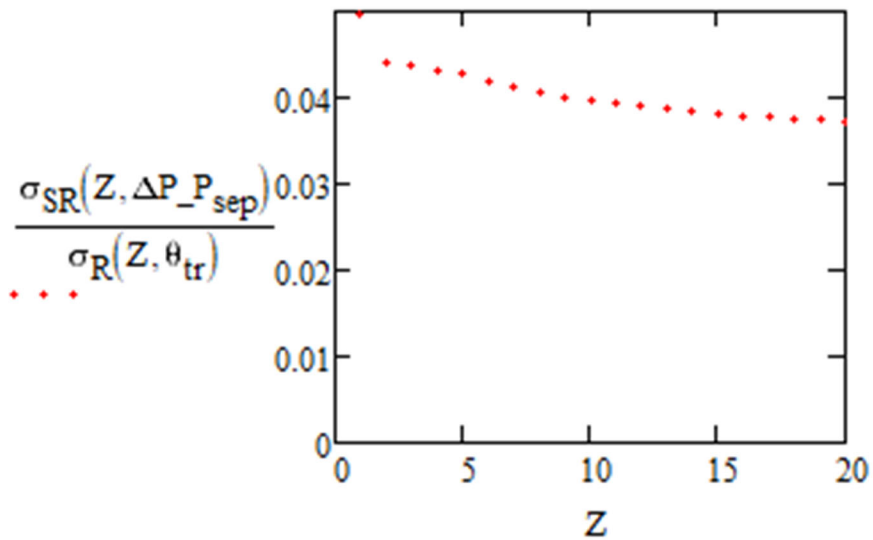
- Later we found out that the statement that the gas scattering is more important is correct at injection energy only
- At the 1 TeV energy the e.-m. scattering was reduced by $(1000/150)^2$~50 times, but e.-m. noise effect was not expected to change much
- Measurements of IBS showed that the magnetic noise contributes much more at the collision energy than the gas scattering

Beam Lifetime in IOTA

- At small intensity the measured beam lifetime is ~175 min
- Other (measured) parameters:
  - acceptances: ε_{xm}=22 μm, ε_{ym}=40 μm, Δp_m=0.27%
  - average β-function: β_{xa}=2.16 m, β_{ya}=1.94 m
- Contributions to lifetime come from
  - elastic gas scattering (discussed here)
  - inelastic scattering on atomic electrons
  - Bremsstrahlung (~4%)







Beam Lifetime in IOTA (2)

- If only elastic scattering is accounted

$$\tau_{gas}^{-1} = \frac{2\pi c r_e^2}{\gamma^2 \beta^3} \left\langle \left(\frac{\beta_x(s)}{\varepsilon_{xm}} + \frac{\beta_y(s)}{\varepsilon_{ym}} \right) n_{eff}(s) \right\rangle_s , \quad n_{eff}(s) = \sum_{\substack{k \\ over\ atoms}} Z_k (Z_k + 1) n_k(s)$$

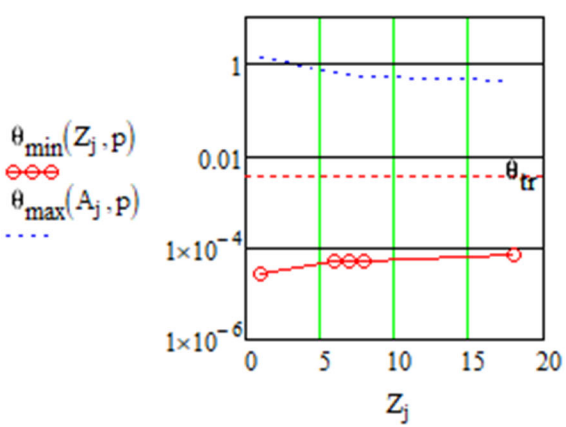
  ⇨ $P_{eff}=4.8\cdot10^{-8}$ Torr of atomic hydrogen equivalent

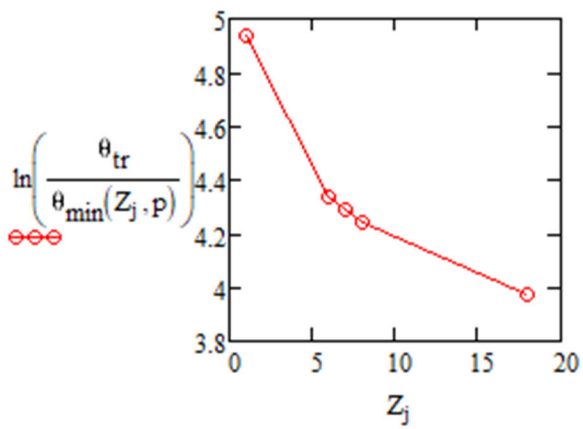
- Accounting of inelastic scattering and bremsstrahlung yields better vacuum ~$4.2\cdot10^{-8}$ Torr (atomic H equivalent)
- The maximum scattering angle is determined by the ring acceptance

$$pmax\cdot10^{10} = \begin{pmatrix} 0 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0 \\ 0.08 \\ 1.097 \\ 0.159 \end{pmatrix} \text{Torr} \quad \begin{pmatrix} H \\ H_2 \\ CO \\ H_2\cdot O \\ C_2H_2 \\ C\cdot H_4 \\ CO_2 \\ Ar \end{pmatrix}$$

$$\sum_j \left[Peff_j \cdot (Z_j + 1) \cdot Z_j \right] = 4.166 \times 10^{-8} \text{ Torr}$$

of atomic hydrogen





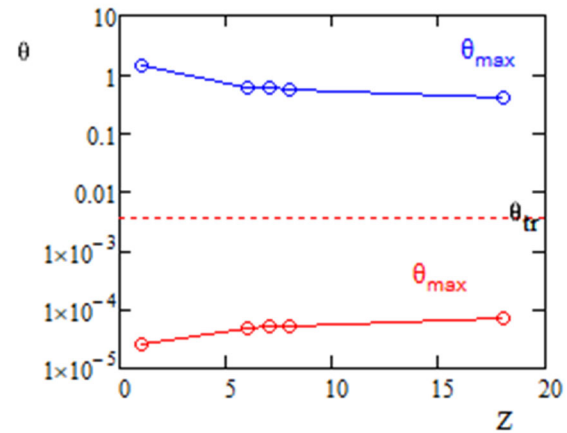
Emittance Growth due to Gas Scattering in IOTA

- Vertical rms emittance set by elastic scattering

$$\bar{I} = \frac{\hat{B}}{2}\ln\left(\frac{4\hat{I}_b}{e}\right), \quad \hat{I}_b \gg 1, \quad \hat{B} \ll 1$$

summing over all gas species and transiting from dimensionless variables one obtains

$$\varepsilon_y = \frac{1}{2}\sum_k \hat{B}_k \left(I_{\min}\right)_k \ln\left(\frac{4I_b}{e\left(I_{\min}\right)_k}\right)$$



- It yields ε_{yGas} = 3.8 nm while measured value is 9 times smaller 0.42 nm

- For perfectly decoupled machine there is a contribution of SR to vertical emittance due to angular spread of radiated photons:

$$\varepsilon_y = \frac{1}{2}\frac{55}{32\sqrt{3}}\frac{\hbar\bar{\beta}_y}{m_e c\rho}$$

That results in $\Delta\varepsilon_y$ = 0.33 pm (negligible in practice)

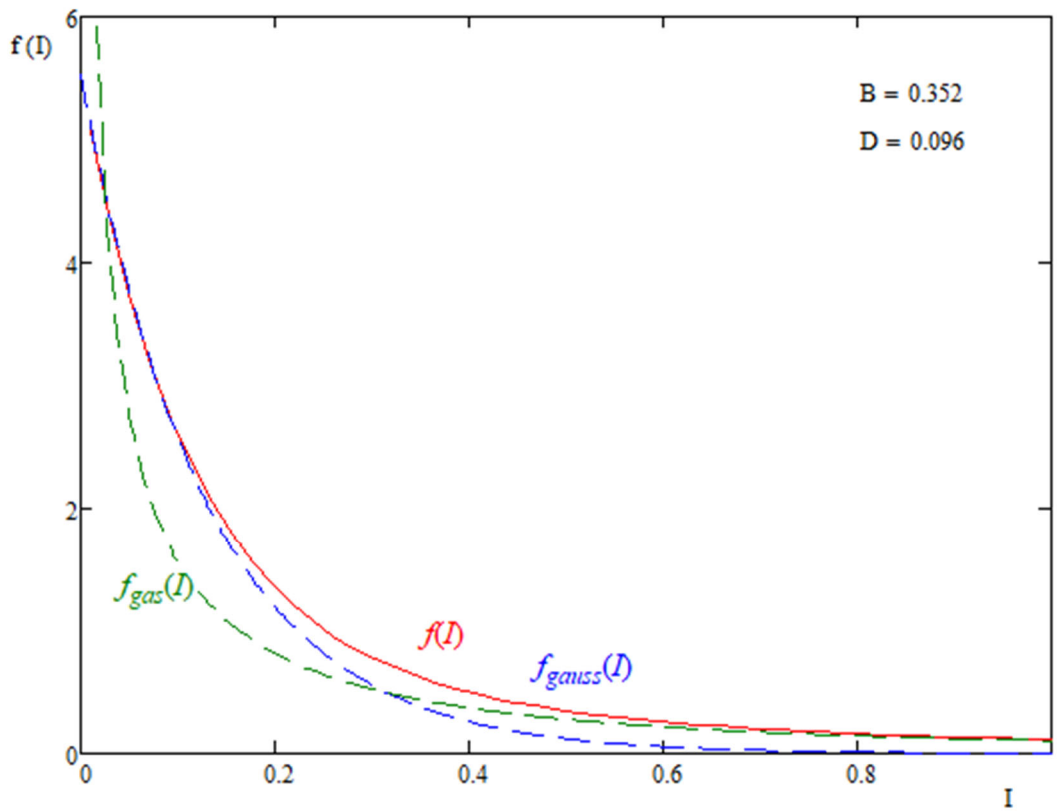
- Measurement show that the major contribution of SR comes from coupling: $\kappa_{xy} \approx 0.5\%$ => ε_{ySR} = 0.25 nm

- I.e. the gas scattering emittance greatly exceeds the measurement
  ⇨ The measurement does not see non-Gaussian tails.
  !!! In the measurements we fit the central bright spot and ignore tails

Equilibrium Vertical Emittance in IOTA

- We add diffusion due to SR

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial \hat{I}}\left(\hat{I} f\right) + D_{SR}\frac{\partial}{\partial \hat{I}}\left(\hat{I}\frac{\partial}{\partial \hat{I}}\right) + \hat{B}\left(\int_0^{I_b} W(\hat{I},\hat{I}')f(\hat{I}',t)\mathrm{d}\hat{I}' - f\right), \quad I < I_b$$



ε_{ySR} = 0.25 nm, I_m = 2.5 nm

Gas scattering increases the Gaussian core width in 1.35 times.

The core includes 72% of particles, However the rms emittance exceeds ε_{ySR} by~20 times

Conclusions

- A study of beam emittance evolution in IOTA included an analysis of IBS and gas scattering
- Observations at small beam intensity exhibited large discrepancy between the measured and predicted vertical beam sizes
  - That forced us to look for a reason
- Further analysis resulted in that the gas scattering creates very large non-Gaussian tails which contain the major fraction of particles
  - These tails were ignored in computation of vertical beam sizes
- That resulted in a further development of mathematical model of gas scattering developed earlier at the Tevatron Run II time
- The model is based on the integrodifferential equation describing particles scattering at the gas in a focusing structure of accelerator