

H^0 Stark Stripping & Element Irradiation

800 MeV Incident H^-

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Overview

- Underlying Concept:
 - “some” fraction of the H^0 emerging from the foil will be in excited states. These excited H^0 can strip in the magnetic field B of the 3rd orbump kicker through the Stark effect.
 - the emergent protons see a deleted net magnetic kick compared to protons emerging from the foil & will track on trajectories different from the nominal.
 - downstream elements *will* be irradiated.
- The point of this study is to determine where these errant protons are lost; how much power is deposited and, most important; is this a concern?
- There are 5 steps in the study:
 - 1) what fraction of the incident H^- emerge from the foil as H^0 ?
 - 2) which excited states are a concern for stripping within orbump3?
 - 3) what is the fractional population of these H^0 excited states?
 - 4) determine the stripping distribution within the magnet for each excited state
 - 5) track these stripped distributions to determine which downstream elements get lit up & how much power is deposited at these locations.

1) H⁰ Fraction Emerging from Foil

The H species decay/generation eqn's are generic examples of those found in some form throughout this study:

$$\frac{dH^-(x)}{dx} = -\sigma_{-0}H^-(x)$$

$$\frac{dH^0(x)}{dx} = +\sigma_{-0}H^-(x) - \sigma_{0+}H^0(x)$$

$$\frac{dH^+(x)}{dx} = +\sigma_{0+}H^0(x)$$

$$H^-(x) = e^{-\sigma_{-0}x}$$

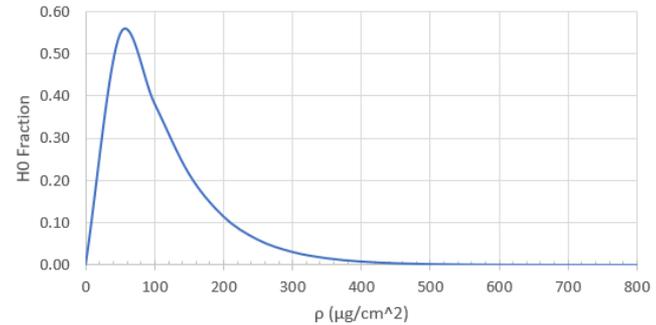
$$H^0(x) = \frac{\sigma_{-0}}{\sigma_{-0} - \sigma_{0+}} [e^{-\sigma_{0+}x} - e^{-\sigma_{-0}x}]$$

$$H^+(x) = \frac{1}{\sigma_{-0} - \sigma_{0+}} [\sigma_{-0}(1 - e^{-\sigma_{0+}x}) - \sigma_{0+}(1 - e^{-\sigma_{-0}x})]$$

$$\sigma_{-0} = 0.676 \cdot 10^{-18} \text{ cm}^2 \quad \sigma_{0+} = 0.264 \cdot 10^{-18} \text{ cm}^2$$

For 17kW incident H⁻ 10W appear as H⁰

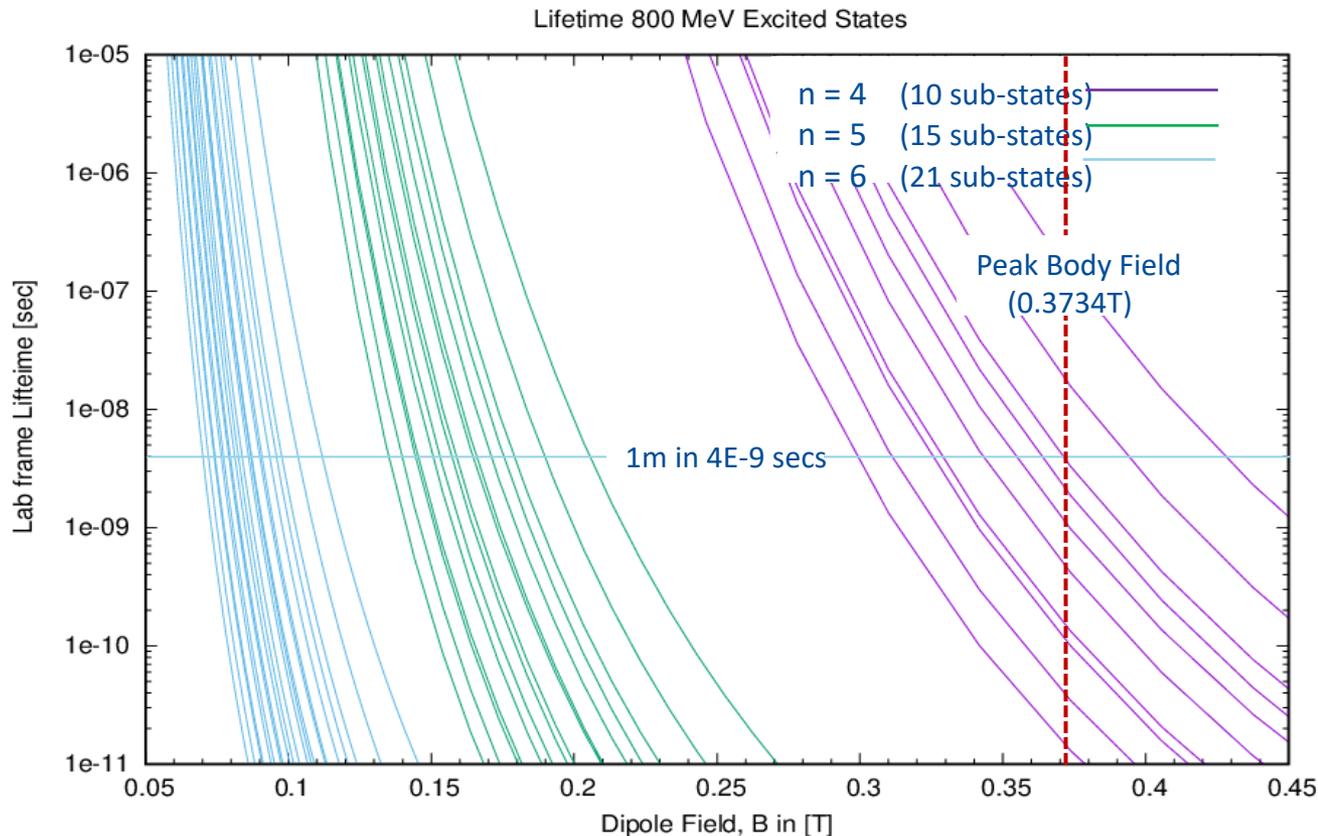
H⁰ Fractional Yield @ 800 MeV



ρ ($\mu\text{g}/\text{cm}^2$)	H ⁰ fraction
100	3.8245E-01
150	2.1623E-01
200	1.1519E-01
250	6.0166E-02
300	3.1214E-02
350	1.6154E-02
400	8.3533E-03
450	4.3182E-03
500	2.2320E-03
550	1.1536E-03
600	5.9626E-04
650	3.0818E-04
700	1.5928E-04
750	8.2325E-05
800	4.2550E-05

2) Which $H^0(n)$ are Relevant? †

- In a field B each principal quantum state n splits into $n(n+1)/2$ sub-states:



† provided by Dave from : W. Chou and A. Drozhdin, *Lifetime of Stark States Hydrogen Atom in Magnetic Field Calculation & Estimation of Losses at Stripping Injection*, (ref??)

3) $H^0(n)$ Excited State Yields[†]

$$h_n(x) = A_1(n)e^{-\sigma_-x} + A_2(n)e^{-\sigma_{12}x} + A_3(n)e^{-\sigma_nx}$$

$$C = \frac{\sigma_{-12}}{\sigma_- - \sigma_{12}}$$

$$A_1(n) = \frac{\sigma_{-n} - C \cdot \sigma_{12n}}{\sigma_{n-} - \sigma_-}$$

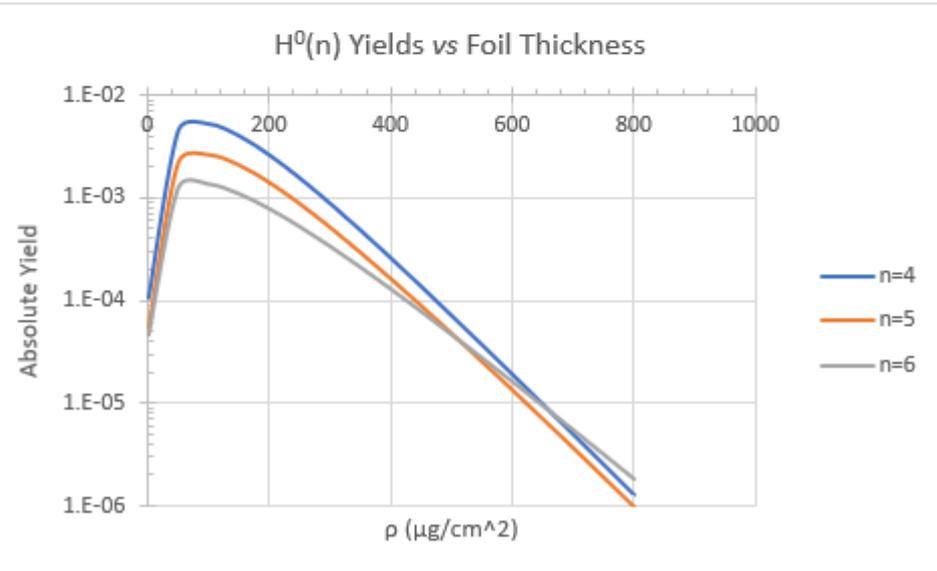
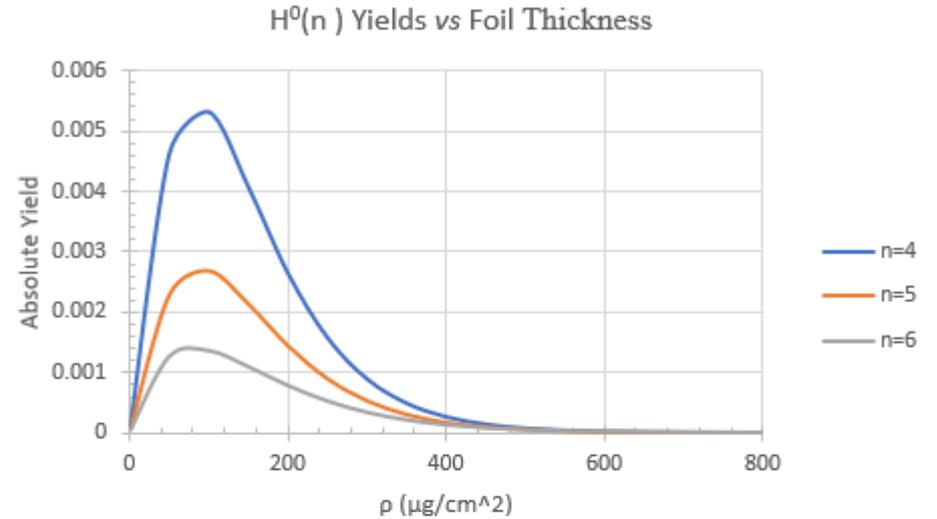
$$A_2(n) = \frac{C \cdot \sigma_{12n}}{\sigma_{n-} - \sigma_{12}}$$

$$A_3(n) = -[A_1(n) + A_2(n)]$$

Designation	Initial State	Final State	Cross -Section (10^{-18} cm ²)
σ_-	–	<u>0,+</u>	0.687±0.016
σ_{-12}	–	1,2	0.668±0.015
σ_{12}	1,2	anything else	0.275±0.010
σ_{-4}	–	4	0.0021±0.0005
σ_{124}	1,2	4	0.0045±0.0010
σ_4	4	anything else	0.369±0.056
σ_{-5}	–	5	0.0011±0.0006
σ_{125}	1,2	5	0.0020±0.0005
σ_5	5	anything else	0.32±0.12
σ_{-6}	–	6	0.00093±0.00055
σ_{126}	1,2	6	0.0006±0.0009
σ_6	6	anything else	0.23±0.16

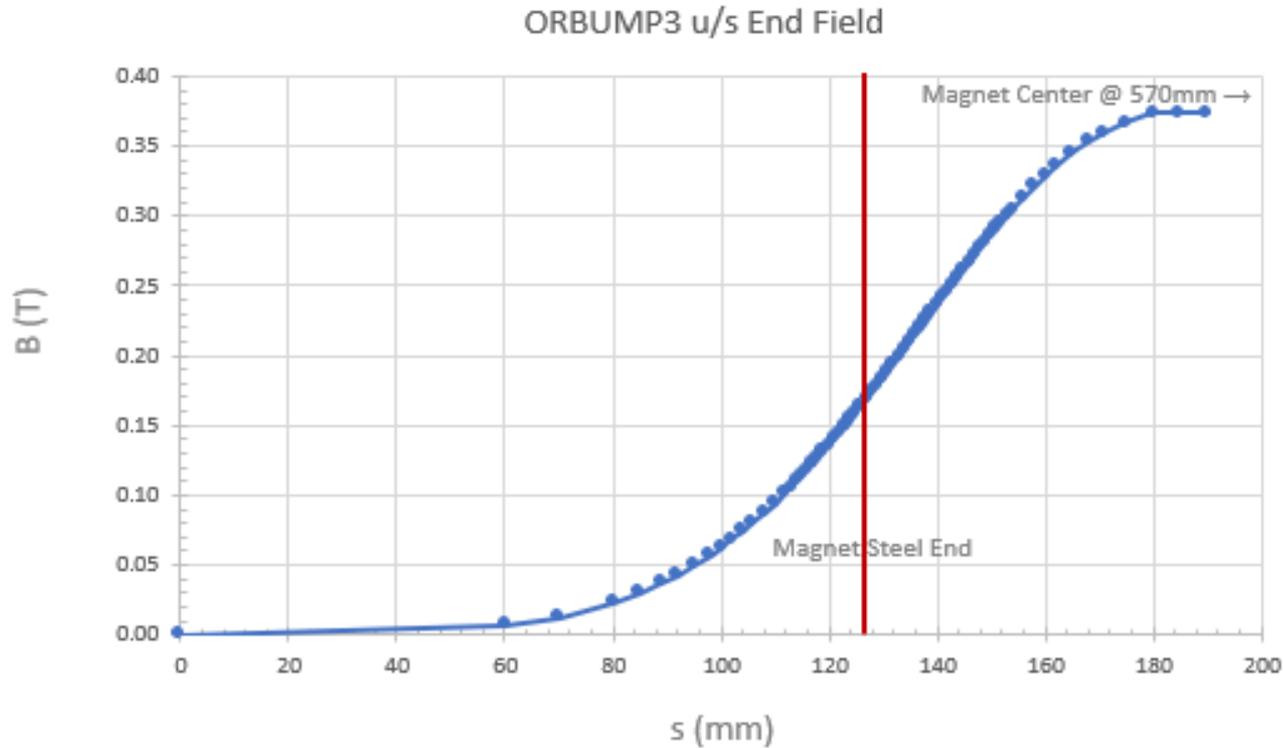
[†] M.S. Gulley *et al*, Measurement of H^- , H^0 , and H^+ yields produced by foil stripping of 800 MeV H^- ions, Phys Rev A 53 (1996)

ρ ($\mu\text{g}/\text{cm}^2$)	n=4	n=5	n=6
1	1.0594E-04	5.5268E-05	4.5938E-05
50	4.6115E-03	2.2801E-03	1.2584E-03
100	5.3037E-03	2.6804E-03	1.3525E-03
150	4.0602E-03	2.1284E-03	1.0872E-03
200	2.6326E-03	1.4374E-03	7.7611E-04
250	1.5654E-03	8.9070E-04	5.1943E-04
300	8.8435E-04	5.2386E-04	3.3389E-04
350	4.8355E-04	2.9765E-04	2.0880E-04
400	2.5863E-04	1.6507E-04	1.2801E-04
450	1.3621E-04	8.9924E-05	7.7323E-05
500	7.0944E-05	4.8328E-05	4.6170E-05
550	3.6648E-05	2.5699E-05	2.7317E-05
600	1.8815E-05	1.3550E-05	1.6044E-05
650	9.6150E-06	7.0959E-06	9.3662E-06
700	4.8960E-06	3.6949E-06	5.4405E-06
750	2.4862E-06	1.9149E-06	3.1469E-06
800	1.2598E-06	9.8847E-07	1.8138E-06



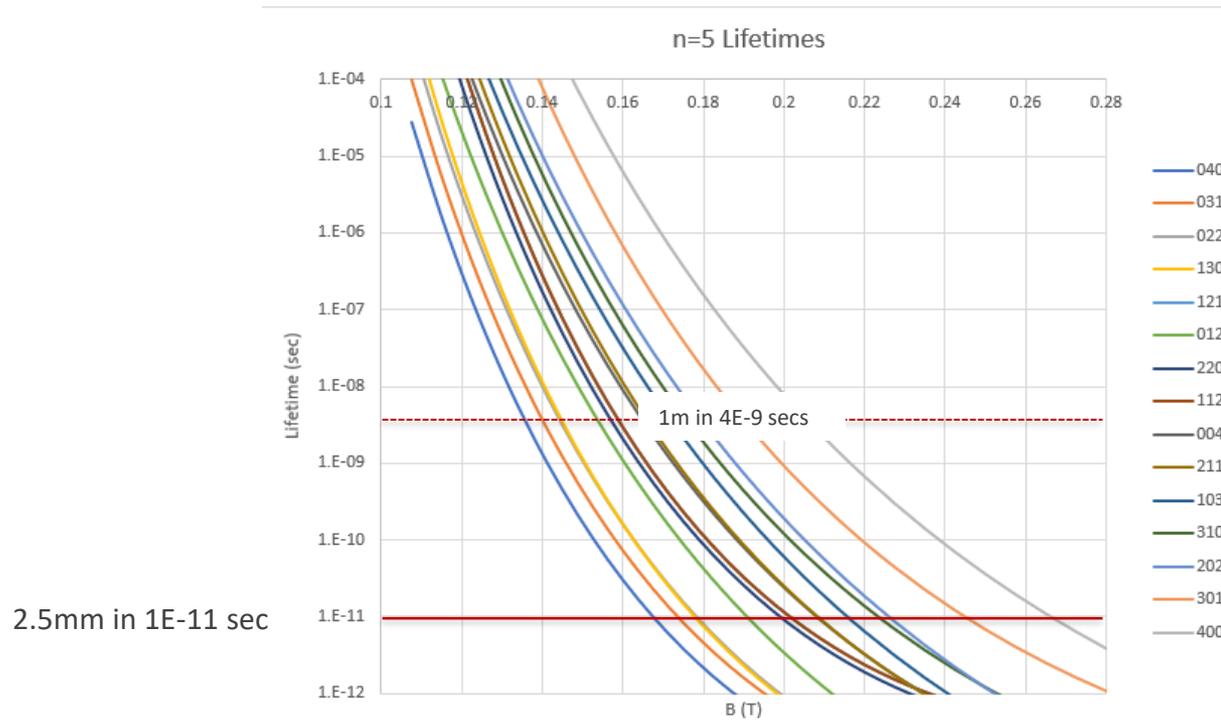
4) $H^0(n)$ Stripping Distribution within Orbump3

End-field of orbump3 modeled by 50 Gauss bin increments[†]



[†] ask Dave where this end-field distribution came from

H⁰ n=5 & 6



- with a peak field of 0.3734T all n=5 & 6 states strip within the end field
- in a first pass we assume a state strips at a lifetime of $\tau = 1E-11$ sec.

H⁰(5) & H⁰(6) tracking model

- the magnet end field is sliced into ~50G bin increments
- each of the 15 individual n=5 sub-states & 21 n=6 sub-states see zero magnetic field until they meet the bin where they encounter their corresponding stripping B field. From that point on, as H⁺, they see the full magnetic field (implementation illustrated on next slide).
- Apertures are assigned to all downstream elements
- 50,000 particles, randomly generated from a Gaussian phase-space distribution (truncated at 99%), are tracked for **each** of the n=5 & n=6 sub-states **from the foil in L11 through to L20** and losses are recorded.

! TOTAL U/S HALF VKICK:ANGLE = -0.03387315

KSLLICE : KICKER, APERTYPE=RECTANGLE, APERTURE= {0.028,0.150} ;

BRHO_800 = 4.881028688 ;

SLICE1 := 0.001 ;
SLICE2 := 0.002 ;
SLICE3 := 0.003 ;
SLICE4 := 0.004 ;
SLICE5 := 0.005 ;
SLICE10 := 0.010 ;
SLICE60 := 0.060 ;

! END FIELDS FROM 0 THROUGH 185 mm

BBAR1 := 3.4066122E-03 *0 ;
BBAR2 := 9.8341465E-03 *0 ;
BBAR3 := 1.7941036E-02 *0 ;
BBAR4 := 2.6585483E-02 *0 ;
BBAR5 := 3.3604615E-02 *0 ;
BBAR6 := 4.0026857E-02 *0 ;
BBAR7 := 4.6322557E-02 *0 ;
BBAR8 := 5.3342184E-02 *0 ;
BBAR9 := 5.9715664E-02 *0 ;
BBAR10 := 6.5247514E-02 *0 ;

BBAR11 := 7.1134090E-02 *0 ; ! No EXCITED STATE STRIPPING: N = PRINCIPLE QUANTUM NUMBER -- NO IDEA WHAT "STATE" MEANS

BBAR12 := 7.7380173E-02 *0 ;
BBAR13 := 8.3988480E-02 *0 ;
BBAR14 := 9.0959455E-02 *0 ; ! N = 6: 8.70E-02 STATE 50, 8.00E-02 STATE 41, 9.00E-02 STATE 140, 9.10E-02 STATE 32
BBAR15 := 9.8291076E-02 *0 ; ! N = 6: 9.30E-02 STATE 131, 9.50E-02 STATE 23, 9.70E-02 STATE 14, 9.80E-02 STATE 122,
BBAR16 := 1.0399083E-01 *0 ; ! N = 6: 1.00E-01 STATE 221, 1.03E-01 STATE 113
BBAR17 := 1.0792286E-01 *0 ; ! N = 6: 1.06E-01 STATE 131, 1.08E-01 STATE 5
BBAR18 := 1.1194154E-01 *0 ; ! N = 6: 1.08E-01 STATE 212
BBAR19 := 1.1604556E-01 *0 ; ! N = 6: 1.13E-01 STATE 50, 1.13E-01 STATE 104
BBAR20 := 1.2023142E-01 *0 ; ! N = 6: 1.19E-01 STATE 203

BBAR21 := 1.2458343E-01 *0 ; ! N = 6: 1.21E-01 STATE 410, 1.23E-01 STATE 302
BBAR22 := 1.2885368E-01 *0 ;
BBAR23 := 1.3328209E-01 *0 ; ! N = 6: 1.31E-01 STATE 401
BBAR24 := 1.3778634E-01 *0 ;
BBAR25 := 1.4236393E-01 *0 ;
BBAR26 := 1.4701214E-01 *0 ; ! N = 6: 1.44E-01 STATE 500
BBAR27 := 1.5172805E-01 *0 ;
BBAR28 := 1.5650851E-01 *0 ;
BBAR29 := 1.6135019E-01 *0 ;
BBAR30 := 1.6624953E-01 *0 ;

BBAR31 := 1.7120277E-01 *0 ; ! N = 5: 1.68E-01 STATE 40
BBAR32 := 1.7620596E-01 *1 ; ! N = 5: 1.75E-01 STATE 31
BBAR33 := 1.8125493E-01 *1 ; ! N = 5: 1.77E-01 STATE 22, 1.78E-01 STATE 130
BBAR34 := 1.8634531E-01 *1 ;
BBAR35 := 1.9147256E-01 *1 ; ! N = 5: 1.87E-01 STATE 121, 1.90E-01 STATE 13
BBAR36 := 1.9663194E-01 *1 ;
BBAR37 := 2.0181851E-01 *1 ; ! N = 5: 1.98E-01 STATE 220, 2.00E-01 STATE 112
BBAR38 := 2.0702717E-01 *1 ;
BBAR39 := 2.1225266E-01 *1 ; ! N = 5: 2.08E-01 STATE 4, 2.08E-01 STATE 211
BBAR40 := 2.1748953E-01 *1 ; ! N = 5: 2.15E-01 STATE 103

BBAR41 := 2.2273129E-01 *1 ;
BBAR42 := 2.2797491E-01 *1 ; ! N = 5: 2.23E-01 STATE 310, 2.25E-01 STATE 202
BBAR43 := 2.3321182E-01 *1 ;
BBAR44 := 2.3843651E-01 *1 ;
BBAR45 := 2.4364406E-01 *1 ;
BBAR46 := 2.4882706E-01 *1 ; ! N = 5: 2.44E-01 STATE 301
BBAR47 := 2.5397958E-01 *1 ;
BBAR48 := 2.5909524E-01 *1 ;
BBAR49 := 2.6416756E-01 *1 ;
BBAR50 := 2.6919003E-01 *1 ; ! N = 5: 2.65E-01 STATE 400

BBAR51 := 2.7415608E-01 *1 ;
BBAR52 := 2.7905912E-01 *1 ;
BBAR53 := 2.8389254E-01 *1 ;
BBAR54 := 2.8864974E-01 *1 ;
BBAR55 := 2.9332413E-01 *1 ;
BBAR56 := 2.9790915E-01 *1 ;
BBAR57 := 3.0239827E-01 *1 ;
BBAR58 := 3.0889612E-01 *1 ;
BBAR59 := 3.1721606E-01 *1 ;
BBAR60 := 3.2503938E-01 *1 ;

BBAR61 := 3.3231879E-01 *1 ;
BBAR62 := 3.4048517E-01 *1 ;
BBAR63 := 3.4906957E-01 *1 ;
BBAR64 := 3.5609266E-01 *1 ;
BBAR65 := 3.6275714E-01 *1 ;
BBAR66 := 3.6987190E-01 *1 ;
BBAR67 := 3.7342929E-01 *1 ;

OB3SLC_1 := KSLLICE, L = SLICE60, VKICK := -BBAR1/BRHO_800 * SLICE60 ;
OB3SLC_2 := KSLLICE, L = SLICE10, VKICK := -BBAR2/BRHO_800 * SLICE10 ;
OB3SLC_3 := KSLLICE, L = SLICE10, VKICK := -BBAR3/BRHO_800 * SLICE10 ;
OB3SLC_4 := KSLLICE, L = SLICES, VKICK := -BBAR4/BRHO_800 * SLICES ;
OB3SLC_5 := KSLLICE, L = SLICE4, VKICK := -BBAR5/BRHO_800 * SLICE4 ;
OB3SLC_6 := KSLLICE, L = SLICE6, VKICK := -BBAR6/BRHO_800 * SLICE6 ;
OB3SLC_7 := KSLLICE, L = SLICE3, VKICK := -BBAR7/BRHO_800 * SLICE3 ;
OB3SLC_8 := KSLLICE, L = SLICES, VKICK := -BBAR8/BRHO_800 * SLICES ;
OB3SLC_9 := KSLLICE, L = SLICE2, VKICK := -BBAR9/BRHO_800 * SLICE2 ;
OB3SLC_10 := KSLLICE, L = SLICE2, VKICK := -BBAR10/BRHO_800 * SLICE2 ;

OB3SLC_11 := KSLLICE, L = SLICE2, VKICK := -BBAR11/BRHO_800 * SLICE2 ;
OB3SLC_12 := KSLLICE, L = SLICE2, VKICK := -BBAR12/BRHO_800 * SLICE2 ;
OB3SLC_13 := KSLLICE, L = SLICE2, VKICK := -BBAR13/BRHO_800 * SLICE2 ;
OB3SLC_14 := KSLLICE, L = SLICE2, VKICK := -BBAR14/BRHO_800 * SLICE2 ;
OB3SLC_15 := KSLLICE, L = SLICE2, VKICK := -BBAR15/BRHO_800 * SLICE2 ;
OB3SLC_16 := KSLLICE, L = SLICE1, VKICK := -BBAR16/BRHO_800 * SLICE1 ;
OB3SLC_17 := KSLLICE, L = SLICE1, VKICK := -BBAR17/BRHO_800 * SLICE1 ;
OB3SLC_18 := KSLLICE, L = SLICE1, VKICK := -BBAR18/BRHO_800 * SLICE1 ;
OB3SLC_19 := KSLLICE, L = SLICE1, VKICK := -BBAR19/BRHO_800 * SLICE1 ;
OB3SLC_20 := KSLLICE, L = SLICE1, VKICK := -BBAR20/BRHO_800 * SLICE1 ;

OB3SLC_21 := KSLLICE, L = SLICE1, VKICK := -BBAR21/BRHO_800 * SLICE1 ;
OB3SLC_22 := KSLLICE, L = SLICE1, VKICK := -BBAR22/BRHO_800 * SLICE1 ;
OB3SLC_23 := KSLLICE, L = SLICE1, VKICK := -BBAR23/BRHO_800 * SLICE1 ;
OB3SLC_24 := KSLLICE, L = SLICE1, VKICK := -BBAR24/BRHO_800 * SLICE1 ;
OB3SLC_25 := KSLLICE, L = SLICE1, VKICK := -BBAR25/BRHO_800 * SLICE1 ;
OB3SLC_26 := KSLLICE, L = SLICE1, VKICK := -BBAR26/BRHO_800 * SLICE1 ;
OB3SLC_27 := KSLLICE, L = SLICE1, VKICK := -BBAR27/BRHO_800 * SLICE1 ;
OB3SLC_28 := KSLLICE, L = SLICE1, VKICK := -BBAR28/BRHO_800 * SLICE1 ;
OB3SLC_29 := KSLLICE, L = SLICE1, VKICK := -BBAR29/BRHO_800 * SLICE1 ;
OB3SLC_30 := KSLLICE, L = SLICE1, VKICK := -BBAR30/BRHO_800 * SLICE1 ;

OB3SLC_31 := KSLLICE, L = SLICE1, VKICK := -BBAR31/BRHO_800 * SLICE1 ;
OB3SLC_32 := KSLLICE, L = SLICE1, VKICK := -BBAR32/BRHO_800 * SLICE1 ;
OB3SLC_33 := KSLLICE, L = SLICE1, VKICK := -BBAR33/BRHO_800 * SLICE1 ;
OB3SLC_34 := KSLLICE, L = SLICE1, VKICK := -BBAR34/BRHO_800 * SLICE1 ;
OB3SLC_35 := KSLLICE, L = SLICE1, VKICK := -BBAR35/BRHO_800 * SLICE1 ;
OB3SLC_36 := KSLLICE, L = SLICE1, VKICK := -BBAR36/BRHO_800 * SLICE1 ;
OB3SLC_37 := KSLLICE, L = SLICE1, VKICK := -BBAR37/BRHO_800 * SLICE1 ;
OB3SLC_38 := KSLLICE, L = SLICE1, VKICK := -BBAR38/BRHO_800 * SLICE1 ;
OB3SLC_39 := KSLLICE, L = SLICE1, VKICK := -BBAR39/BRHO_800 * SLICE1 ;
OB3SLC_40 := KSLLICE, L = SLICE1, VKICK := -BBAR40/BRHO_800 * SLICE1 ;

OB3SLC_41 := KSLLICE, L = SLICE1, VKICK := -BBAR41/BRHO_800 * SLICE1 ;
OB3SLC_42 := KSLLICE, L = SLICE1, VKICK := -BBAR42/BRHO_800 * SLICE1 ;
OB3SLC_43 := KSLLICE, L = SLICE1, VKICK := -BBAR43/BRHO_800 * SLICE1 ;
OB3SLC_44 := KSLLICE, L = SLICE1, VKICK := -BBAR44/BRHO_800 * SLICE1 ;
OB3SLC_45 := KSLLICE, L = SLICE1, VKICK := -BBAR45/BRHO_800 * SLICE1 ;
OB3SLC_46 := KSLLICE, L = SLICE1, VKICK := -BBAR46/BRHO_800 * SLICE1 ;
OB3SLC_47 := KSLLICE, L = SLICE1, VKICK := -BBAR47/BRHO_800 * SLICE1 ;
OB3SLC_48 := KSLLICE, L = SLICE1, VKICK := -BBAR48/BRHO_800 * SLICE1 ;
OB3SLC_49 := KSLLICE, L = SLICE1, VKICK := -BBAR49/BRHO_800 * SLICE1 ;
OB3SLC_50 := KSLLICE, L = SLICE1, VKICK := -BBAR50/BRHO_800 * SLICE1 ;

OB3SLC_51 := KSLLICE, L = SLICE1, VKICK := -BBAR51/BRHO_800 * SLICE1 ;
OB3SLC_52 := KSLLICE, L = SLICE1, VKICK := -BBAR52/BRHO_800 * SLICE1 ;
OB3SLC_53 := KSLLICE, L = SLICE1, VKICK := -BBAR53/BRHO_800 * SLICE1 ;
OB3SLC_54 := KSLLICE, L = SLICE1, VKICK := -BBAR54/BRHO_800 * SLICE1 ;
OB3SLC_55 := KSLLICE, L = SLICE1, VKICK := -BBAR55/BRHO_800 * SLICE1 ;
OB3SLC_56 := KSLLICE, L = SLICE1, VKICK := -BBAR56/BRHO_800 * SLICE1 ;
OB3SLC_57 := KSLLICE, L = SLICE1, VKICK := -BBAR57/BRHO_800 * SLICE1 ;
OB3SLC_58 := KSLLICE, L = SLICE2, VKICK := -BBAR58/BRHO_800 * SLICE2 ;
OB3SLC_59 := KSLLICE, L = SLICE2, VKICK := -BBAR59/BRHO_800 * SLICE2 ;
OB3SLC_60 := KSLLICE, L = SLICE2, VKICK := -BBAR60/BRHO_800 * SLICE2 ;

OB3SLC_61 := KSLLICE, L = SLICE2, VKICK := -BBAR61/BRHO_800 * SLICE2 ;
OB3SLC_62 := KSLLICE, L = SLICES, VKICK := -BBAR62/BRHO_800 * SLICES ;
OB3SLC_63 := KSLLICE, L = SLICES, VKICK := -BBAR63/BRHO_800 * SLICES ;
OB3SLC_64 := KSLLICE, L = SLICES, VKICK := -BBAR64/BRHO_800 * SLICES ;
OB3SLC_65 := KSLLICE, L = SLICES, VKICK := -BBAR65/BRHO_800 * SLICES ;
OB3SLC_66 := KSLLICE, L = SLICES, VKICK := -BBAR66/BRHO_800 * SLICES ;
OB3SLC_67 := KSLLICE, L = SLICES, VKICK := -BBAR67/BRHO_800 * SLICES ;

ORB3_ENDFLD : LINE = (OB3SLC_1 , OB3SLC_2 , OB3SLC_3 , OB3SLC_4 , OB3SLC_5 , OB3SLC_6 , OB3SLC_7 , OB3SLC_8 , OB3SLC_9 ,
OB3SLC_10 , OB3SLC_11 , OB3SLC_12 , OB3SLC_13 , OB3SLC_14 , OB3SLC_15 , OB3SLC_16 , OB3SLC_17 , OB3SLC_18 , OB3SLC_19 ,
OB3SLC_20 , OB3SLC_21 , OB3SLC_22 , OB3SLC_23 , OB3SLC_24 , OB3SLC_25 , OB3SLC_26 , OB3SLC_27 , OB3SLC_28 , OB3SLC_29 ,
OB3SLC_30 ,
MULBUMP_3 ,
OB3SLC_31 , OB3SLC_32 , OB3SLC_33 , OB3SLC_34 , OB3SLC_35 , OB3SLC_36 , OB3SLC_37 , OB3SLC_38 , OB3SLC_39 ,
OB3SLC_40 , OB3SLC_41 , OB3SLC_42 , OB3SLC_43 , OB3SLC_44 , OB3SLC_45 , OB3SLC_46 , OB3SLC_47 , OB3SLC_48 , OB3SLC_49 ,
OB3SLC_50 , OB3SLC_51 , OB3SLC_52 , OB3SLC_53 , OB3SLC_54 , OB3SLC_55 , OB3SLC_56 , OB3SLC_57 , OB3SLC_58 , OB3SLC_59 ,
OB3SLC_60 , OB3SLC_61 , OB3SLC_62 , OB3SLC_63 , OB3SLC_64 , OB3SLC_65 , OB3SLC_66 , OB3SLC_67) ;

! U/S BODY KICK
LBODY_US := 0.38500 ;
VK_US := 0.0677463 * LBODY_US/0.8855 ;
ORB3BOD_US : VKICKER, L = LBODY_US, KICK := -VK_US, APERTYPE=RECTANGLE, APERTURE= {0.028,0.150} ;

! 1st HALF OF KICKER MAGNET :
ORB3_US : LINE = (ORB3_ENDFLD , ORB3BOD_US) ;



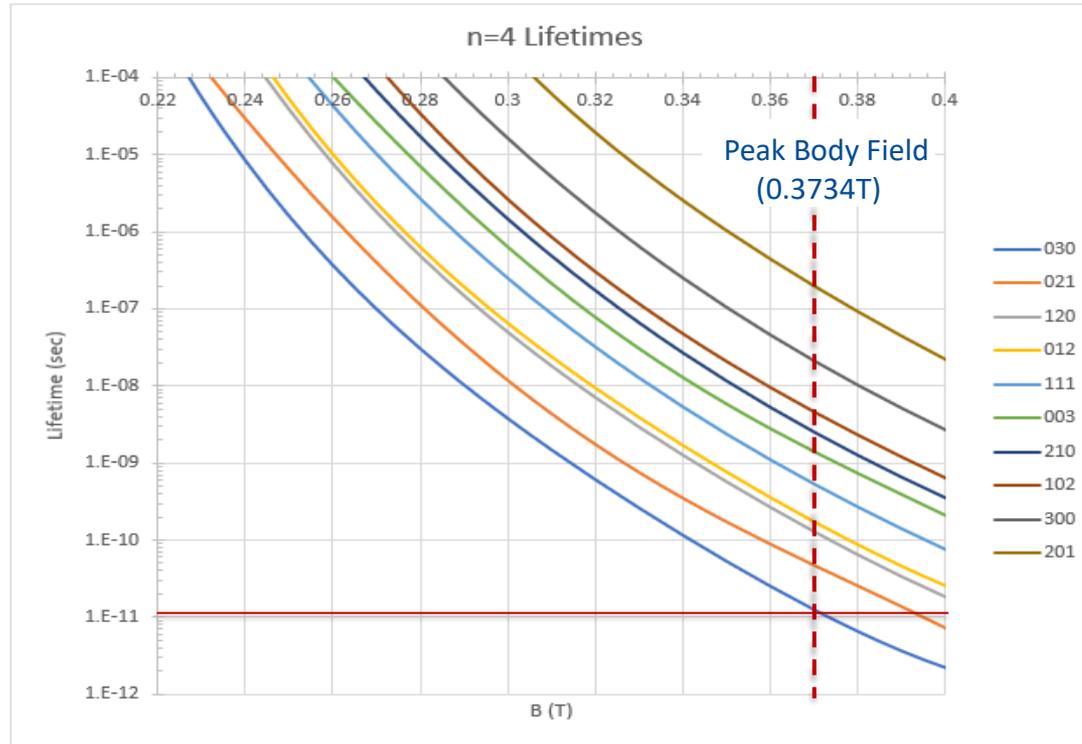
n=5 & 6 loss distributions



- For a 17 kW incident H⁻ beam, power deposited on 1st notcher kicker:

$$17 \cdot 10^3 * 1.36 \cdot 10^{-5} [h^0(n = 5) \text{ fraction}] * 28/750 = 9 \text{ mW}$$

H⁰ n=4 tracking



- with a $\tau = 1\text{E-}11$ sec stripping criterion, as in n=5 tracking, the conclusion would be that no n=4 states would strip, but that isn't right . . .
- since τ is finite in all magnetic fields B some fraction of the H⁰(4) states *will* strip within the magnet.
- A different tracking approach is needed to capture this process.

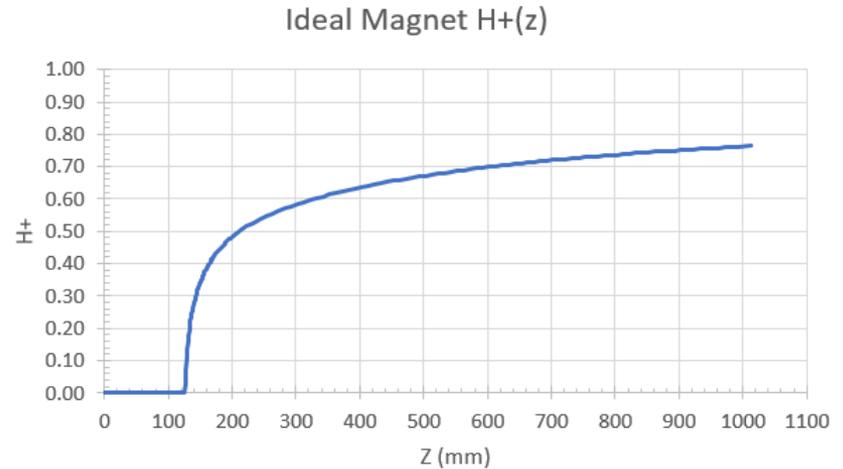
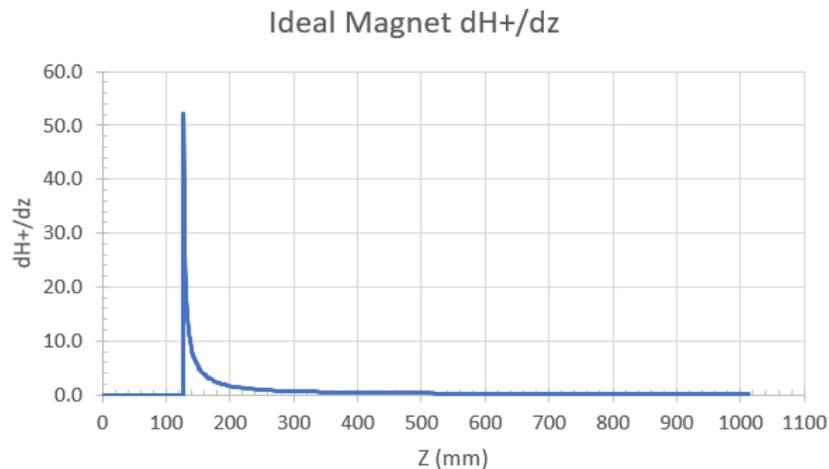
n=4 stripping in the 'ideal' magnet

First, consider an ideal magnet to see what to expect:

- in an ideal magnet (no end-fields) $B=0.3734\text{T}$ constant across 0.8855m
- the stripping equations can then be solved analytically for each sub-state

$$\frac{dh^0(z)}{dz} = -\frac{1}{\tau \cdot \beta c} \cdot h^0(z)$$

$$\frac{dH^+}{dz} = +\frac{1}{\tau \cdot \beta c} \cdot h^0(z) = \frac{1}{\tau \cdot \beta c} e^{-z/\tau \cdot \beta c} \rightarrow H^+(z) = \left(1 - e^{-z/\tau \cdot \beta c}\right) \quad (1)$$



- 76% of the initial $h^0(4)$ strip in the ideal orbump3

implementing the variation of τ with $B(z)$

- recognize that stripping eqn's (1) are valid – even for very thin slices Δz – provided that τ is a constant over that range, *i.e.*: the probability of h^0 stripping is simply proportional to the # of h^0 present in the Δz slice.
- so, the limit can be taken that the slices are infinitesimal, $\Delta z \rightarrow dz$, and $\tau \rightarrow \tau(z)$

$$\frac{dh^0}{dz} = -\frac{1}{\tau(z) \cdot \beta c} \cdot h^0(z) \tag{2}$$

$$\frac{dH^+}{dz} = +\frac{1}{\tau(z) \cdot \beta c} \cdot h^0(z)$$

- which, for *each* of the 10 sub-states, have solutions of the form:

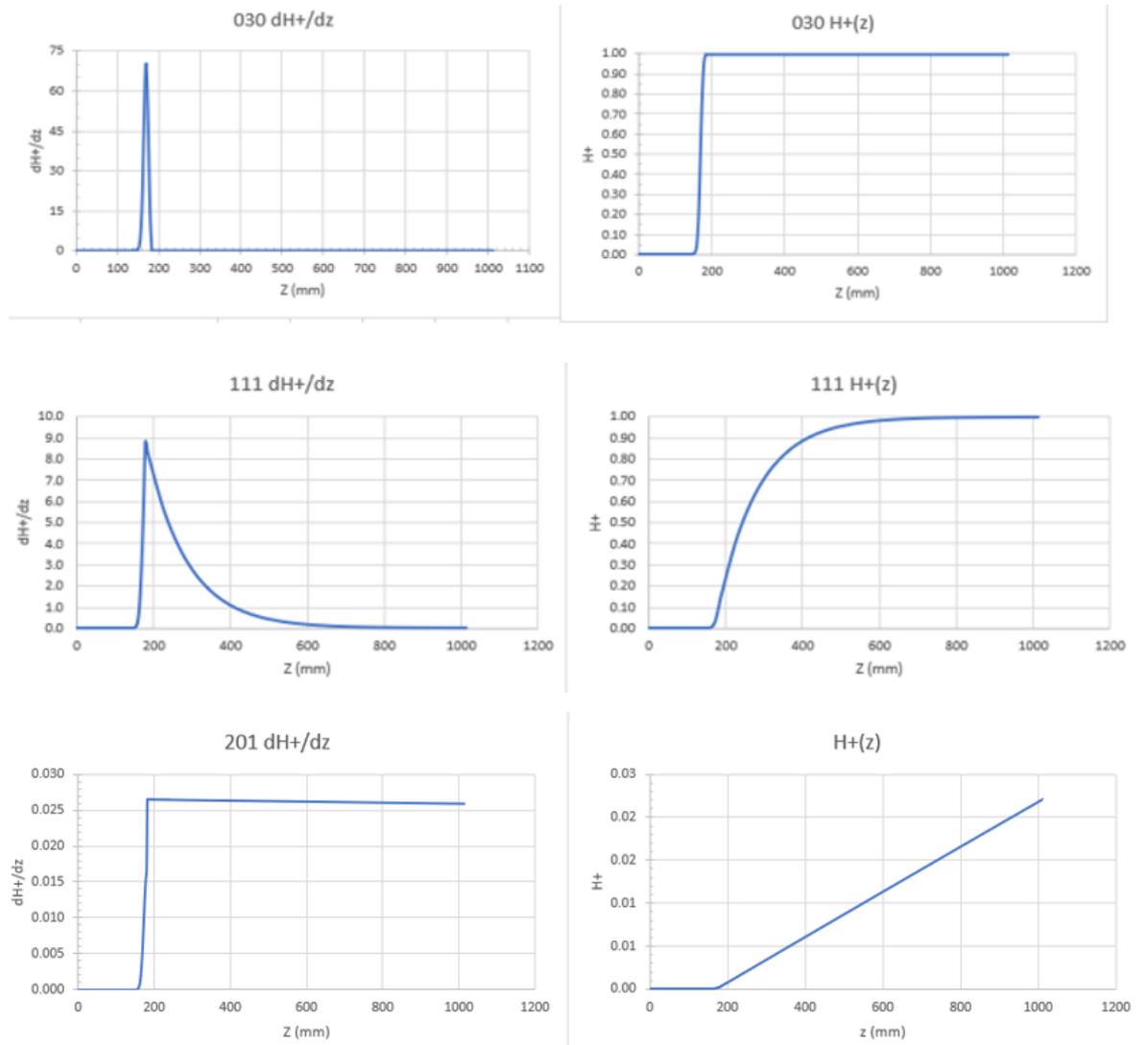
$$\begin{aligned} h^0(z) &= e^{-\int_0^z dz' / \tau(z') \cdot \beta c} \\ H^+(z) &= (1 - e^{-\int_0^z dz' / \tau(z') \cdot \beta c}) \end{aligned} \tag{3}$$

- solving these equations accurately in *Excel* is a bit challenging, but not impossible.

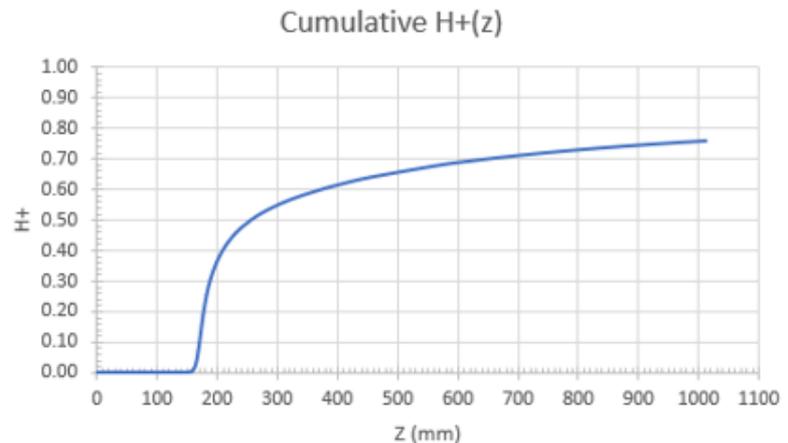
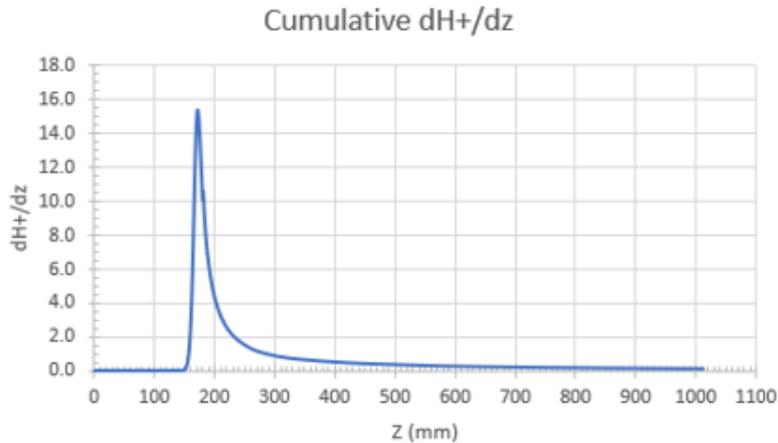
H⁰(4) tracking model

- H⁺ distributions from the individual sub-states are not tracked, as they were for h⁰ n=5 & 6. For n=4 only the sum of the 10 sub-state contributions to H⁺ generation is tracked.
- for n=4 the magnetic field is sliced further into a sufficient number of *B*-field bins such that:
 - integration of *B* gives the correct kick angle;
 - accurate integration of eqn's (3) is possible, and;
 - 5% increments in the H⁺ population can be identified (160 slices).
- 50,000 particles, randomly generated from a Gaussian phase-space distribution (truncated at 99%), are tracked through each of the 5% H⁺ increments from the foil in L11 through to L20 and losses are recorded.

$H^0(4) \rightarrow H^+$ distribution sampler from the $n(n+1)/2=10$ sub-states



cumulative $H^0(4) \rightarrow H^+$ distributions



- 76% of the initial $H^0(4)$ strip in orbump3 (same as 'ideal' magnet!)

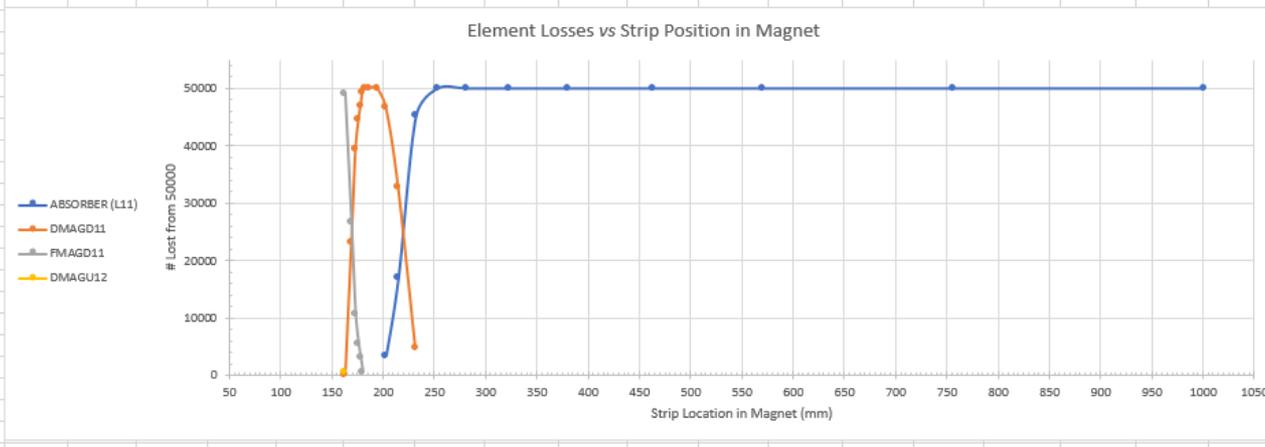
Two variations of the L11 absorber model were studied:

- a) ABS has same aperture as the notcher absorber – $2\frac{5}{8}$ " diameter
- b) ABS a drift – the same 3" diameter as the beam pipe.

n=4 loss distribution (a)

(a) ABS : COLLIMATOR , L = 0.6000 , APERTYPE = CIRCLE, APERTURE = {0.0333375}; ! L11 absorber

N = 4																				SUM OF ALL 10 STATES (Total Stripping Fraction Normalized to 100%)	
Δ % losses	95-100	90-95	85-90	80-85	75-80	70-75	65-70	60-65	55-60	50-55	45-50	40-45	35-40	30-40	25-30	20-25	15-20	10-15	5-10	0-5	
Distance into Magnet	1001	756	570	464	381	323	282	253	232	215	203	194	187	183	180	178	176	173	169	163	
ABSORBER (L11)	50000	50000	50000	50000	50000	50000	50000	50000	45194	17112	3406										465,712
DMAGD11									4806	32888	46594	50000	50000	50000	49358	46955	44559	39322	23217	189	437,888
FMAGD11															642	3045	5441	10678	26783	49151	95,740
DMAGU12																				660	660
lost	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	1,000,000



- For a 17 kW incident H^- beam, power deposited on L11 absorber:

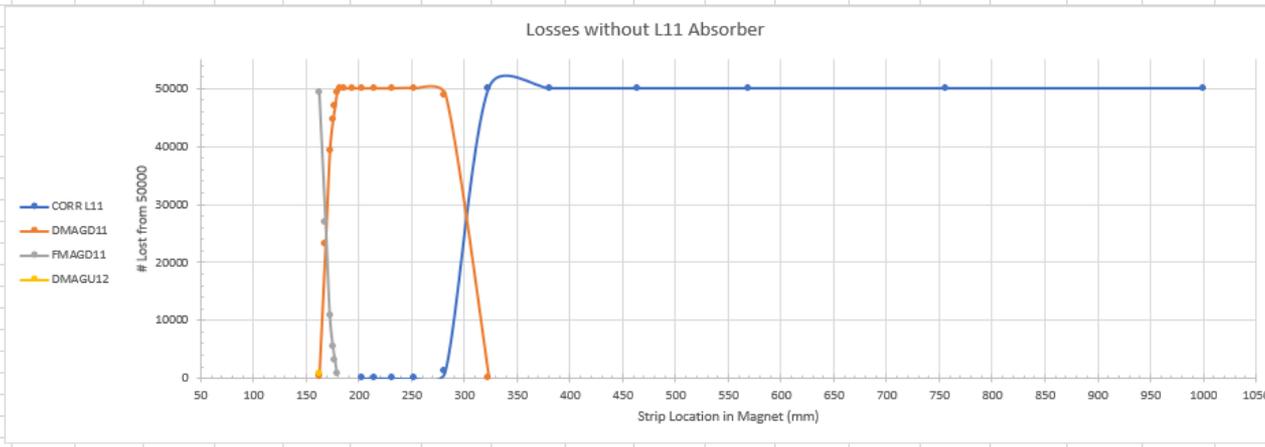
$$17 \cdot 10^3 * 1.88 \cdot 10^{-5} [h^0(n=4) \text{ fraction}] * \frac{466}{1000} (\text{absorber losses}) * 0.76 (\text{total initial } h^0 \text{ stripping fraction}) = 113 \text{ mW}$$

n=4 loss distribution (b)

(b) ABS : DRIFT , L = 0.6000 ;

! L11 absorber

	N = 4																			SUM OF ALL 10 STATES (Total Stripping Fraction Normalized to 100%)	
Δ % losses	95-100	90-95	85-90	80-85	75-80	70-75	65-70	60-65	55-60	50-55	45-50	40-45	35-40	30-40	25-30	20-25	15-20	10-15	5-10	0-5	
Distance into Magnet	1001	756	570	464	381	323	282	253	232	215	203	194	187	183	180	178	176	173	169	163	
CORR L11	50000	50000	50000	50000	50000	50000	1169														301,169
DMAGD11							48831	50000	50000	50000	50000	50000	50000	50000	49358	46955	44559	39322	23217	189	503,600
FMAGD11															642	3045	5441	10678	26783	49151	95,740
DMAGU12																				660	660
lost	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	50000	1,000,000



- For a 17 kW incident H^- beam, power deposited on L11 corrector package:

$$17 \cdot 10^3 * 1.88 \cdot 10^{-5} [h^0(n = 4) \text{ fraction}] * \frac{300}{1000} (\text{corrector losses}) * 0.76 (\text{total initial } h^0 \text{ stripping fraction}) = 72 \text{ mW}$$

H⁰(5) re-visited

- as of 2 weeks ago:

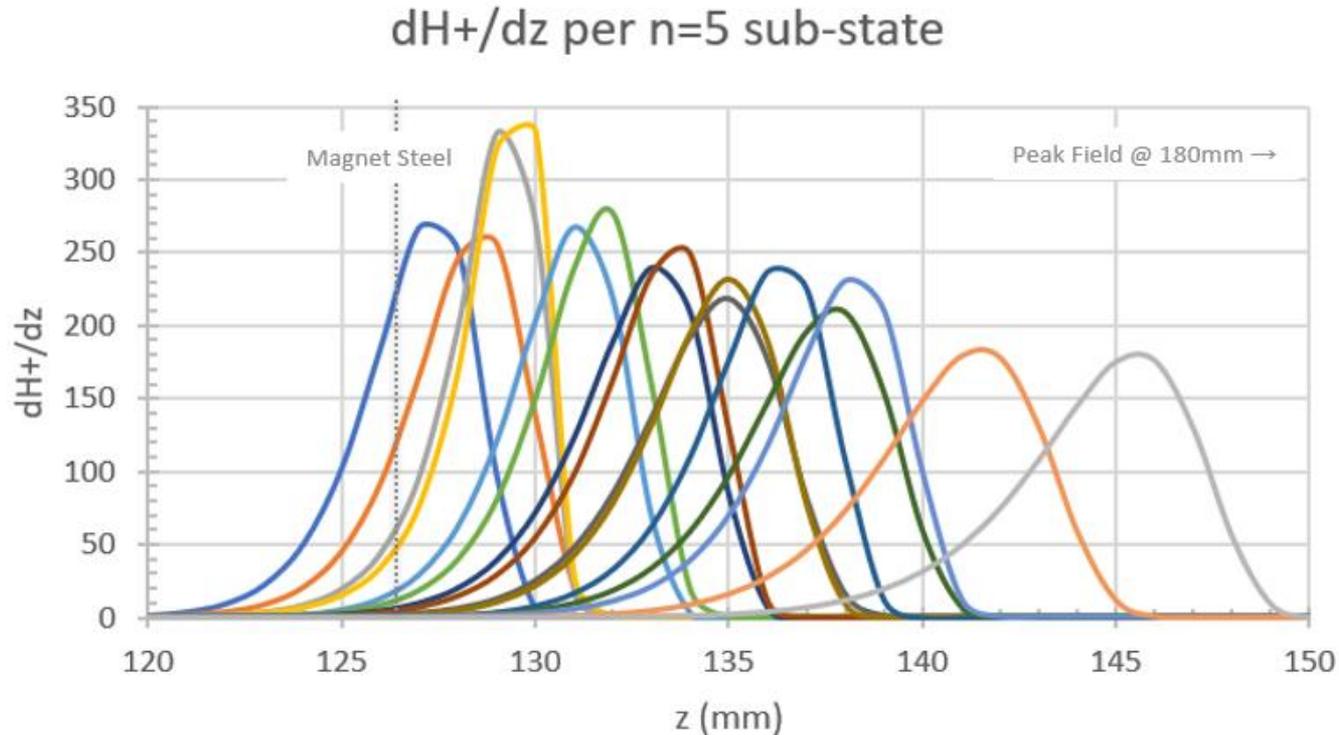
The analysis of H⁰(5) stripping reported here assumed that all sub-states stripped instantly in the field B at which their lifetimes became exactly $\tau = 1\text{E-}11$ seconds and converted to H⁺. This might be overly simplified & the n=5 analysis should be repeated using the more precise formalism employed in the H⁰(4) studies, which accounts for the variation of τ with B . It is not expected that the re-analysis of n=5 will qualitatively alter the conclusions, but this needs to be verified.

♪ done!

1. the variation of τ with field B is known;
2. the variation of B with position z through the magnet is known, so:
3. a 'local' lifetime τ can be assigned to all positions z in the magnet, and the H⁰ decay / H⁺ generation equations can be solved for each of the 15 n=5 sub-states, as they were for n=4:

$$h^0(z) = e^{-\int_0^z dz' / \tau(z') \cdot \beta c}$$
$$H^+(z) = (1 - e^{-\int_0^z dz' / \tau(z') \cdot \beta c})$$

$H^0(5) \rightarrow H^+$ distributions from the $n(n+1)/2=15$ sub-states



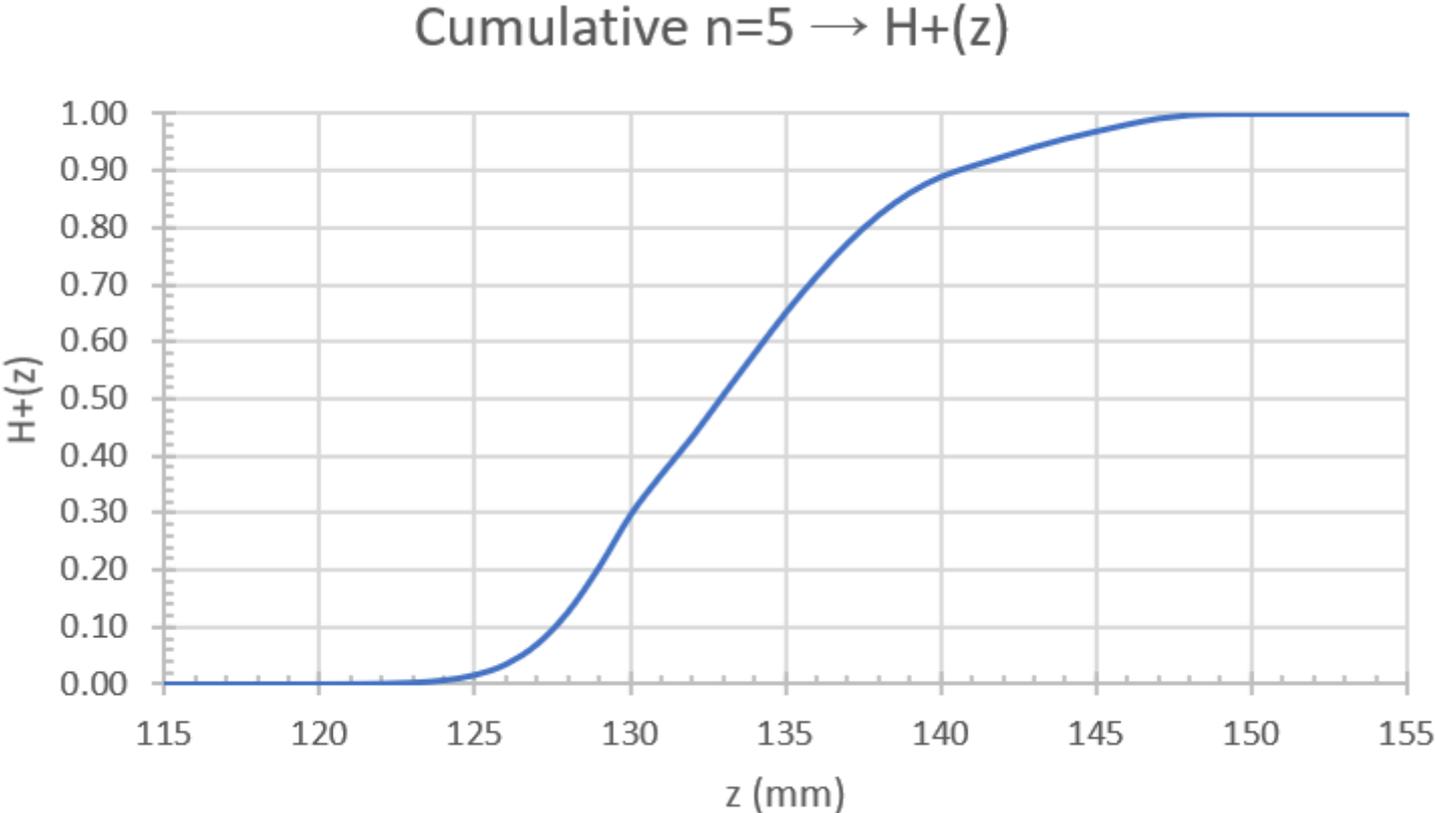
- peak field of 0.3734T reached at $z = 180$ mm
- all 15 sub-states strip in a 30mm range from $z = 120$ -150mm ($B = 0.136$ -0.290T)

rough comparison of $\tau = 1\text{E-}11$ sec vs $\tau \rightarrow \tau(z)$ approaches

n=5 sub-state	$\tau = 1\text{E-}11$ sec (kG)	dH^+ / dz Peak (kG)	$\langle B \rangle$ (kG)
040	1.68	1.70	1.684
031	1.75	1.80	1.746
022	1.77	1.80	1.768
130	1.78	1.83	1.770
121	1.87	1.91	1.876
013	1.90	1.96	1.905
220	1.98	2.01	1.975
112	2.00	2.06	1.992
004	2.08	2.12	2.073
211	2.08	2.12	2.076
103	2.15	2.17	2.144
310	2.23	2.28	2.215
202	2.25	2.28	2.240
301	2.44	2.49	2.410
400	2.65	2.70	2.613

- so, the $\tau = 1\text{E-}11$ sec guesstimate was probably pretty good!

cumulative $H^0(5) \rightarrow H^+$ distribution



n=5 loss distribution



- For a 17 kW incident H⁻ beam, power deposited on 1st notcher kicker:

$$17 \cdot 10^3 * 1.36 \cdot 10^{-5} [h^0(n = 5) \text{ fraction}] * 2^8 / 1,000 = 7 \text{ mW}$$

Summary

- H^0 emerging from the foil in excited states $n=4,5$ & 6 can strip in orbump3 due to the Stark effect. For $H^0 \rightarrow e + H^+$ within the magnet the resulting H^+ experience a depleted kick relative to the nominal circulating proton beam and can result in irradiation of downstream elements.
- Loss distributions of the H^+ from the $n(n+1)/2$ H^0 sub-states in each principal quantum state n were tracked from the foil through to L20.
 - $n = 6$:
 - 100% strip within the end-field:
 - 100% of the H^+ enter the circulating beam – **272 mW**.
 - $n = 5$:
 - 100% strip within the end-field:
 - 12% wallop gradient magnets, but 4% hit the 1st notcher kicker, depositing **7 mW**, 84% enter the circulating beam – **194 mW**.
 - $n = 4$:
 - 76% strip within orbump3 & 65% of these occur within the peak body field of the magnet:
 - 2 scenarios were studied:
 - 1) the L11 absorber has a circular aperture of $2\frac{5}{8}$ " (same as the notcher absorber); 35% of initial $H^0(4)$ hit absorber, depositing **113 mW**;
 - 2) the L11 absorber behaves like a drift, *i.e.*: the same aperture as the beam pipe; 23% of initial $H^0(4)$ hit the L11 corrector package, depositing **72 mW**.

summary (cont'd)

- Although the # of particles tracked did not provide exhaustive statistics, no red-flag issues emerged that would indicate more detailed tracking was warranted.

In particular:

- the majority of particles were lost on gradient magnets.

Otherwise:

- a small fraction of the lost $n=5$ H^0 dinged the 1st notcher kicker, and;
- 23-35% of the $n=4$ H^0 lit up the L11 absorber/corrector region.

- **N.B:** no other components were irradiated. (*your RF is safe, Tan!*).

Looking Forward

1. For each of the $n=4, 5, 6$ states a fraction of the H^+ enter the circulating beam:
 - $n = 4$: 24%
 - $n = 5$: 83%
 - $n = 6$: 100%

The current study did not address the fate of these H^+ . Are they lost somewhere in the machine other than revealed by the $L11 \rightarrow L20$ tracking? To what extent does this lead to emittance growth? How much ends up in halo? This would be a useful study.

2. It would be worthwhile to track the H^+ generated by the 'ideal' magnet to get a sense of the importance of the shape of the end-field B distribution.
3. The L11 region losses from $H^0(4)$ stripping should spark discussion of optimal absorber design.

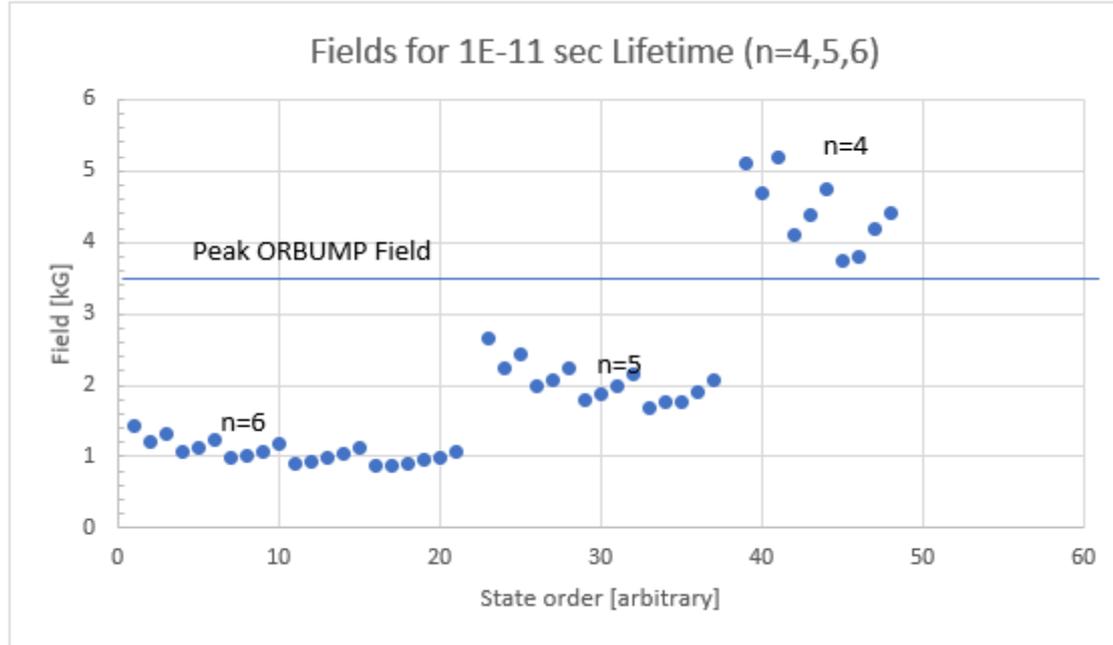
The formalism & techniques being developed here are directly applicable to BAR lattice design, in which the permanent magnets will be particularly sensitive to radiation.

*“Now this not the end. It is not even the beginning of the end.
But it is, perhaps, the end of the beginning.”*



Backoff Slides





† compiled by Dave

more detail on the generation of h_n fractions

- from the *Gunney* paper[†] & references therein:

$$\begin{aligned}\frac{dH^-(x)}{dx} &= -\sigma_- \cdot H^-(x) \\ \frac{dh_{12}(x)}{dx} &= +\sigma_{-12} \cdot H^-(x) - \sigma_{12} \cdot h_{12}(x) \\ \frac{dh_n(x)}{dx} &= +\sigma_{-n} \cdot H^-(x) - \sigma_n \cdot h_n(x) + \sigma_{12n} \cdot h_{12}(x)\end{aligned}$$

$$H^-(x) = e^{-\sigma_- x}$$

$$h_{12}(x) = C \cdot (e^{-\sigma_{12}x} - e^{-\sigma_- x})$$

$$h_n(x) = A_1(n)e^{-\sigma_- x} + A_2(n)e^{-\sigma_{12}x} + A_3(n)e^{-\sigma_n x}$$

$$C = \frac{\sigma_{-12}}{\sigma_- - \sigma_{12}}$$

$$A_1(n) = \frac{\sigma_{-n} - C \cdot \sigma_{12n}}{\sigma_{n-} - \sigma_-}$$

$$A_2(n) = \frac{C \cdot \sigma_{12n}}{\sigma_{n-} - \sigma_{12}}$$

$$A_3(n) = -[A_1(n) + A_2(n)]$$

[†] M.S. Gulley *et al*, Measurement of H^- , H^0 , and H^+ yields produced by foil stripping of 800 MeV H^- ions, Phys Rev A 53 (1996)

Here's the apertures I've included.

```
BRF      : INSTRUMENT      , L = 2.35      , APERTYPE = CIRCLE, APERTURE = {0.0381} ;

NOTCH_ABS : COLLIMATOR      , L = 2.743    , APERTYPE = CIRCLE, APERTURE = {0.0333375} ;      ! Notcher absorber
NOTCHER   : HKICKER        , L = 0.54     , KICK = 0, APERTYPE=RECTANGLE, APERTURE = {0.0345,0.0325} ;      ! Notcher kicker at L12

MKS02    : VKICKER        , L = 1.080    , KICK = 0, APERTYPE=RECTANGLE, APERTURE = {0.0325,0.0345} ;
MKS12    : VKICKER        , L = 1.08     , KICK = 0, APERTYPE=RECTANGLE, APERTURE = {0.0325,0.0345} ;      ! vertical extraction kicker
```

!----- Corrector Package Elements -----

```
DHZ      : HKICKER        , L = 0.024    , CALIB = 0.000366 / 0.024 / Brho , APERTYPE = CIRCLE, APERTURE = {0.060325} ;
DVT      : VKICKER        , L = 0.024    , CALIB = 0.000365 / 0.024 / Brho , APERTYPE = CIRCLE, APERTURE = {0.060325} ;
NQAD     : QUADRUPOLE     , L = 0.024    , CALIB = 0.002489 / 0.024 / Brho , APERTYPE = CIRCLE, APERTURE = {0.060325} ;
SQAD     : QUADRUPOLE     , L = 0.024    , CALIB = -0.003924 / 0.024 / Brho , APERTYPE = CIRCLE, APERTURE = {0.060325} ;
NSEXT    : SEXTUPOLE     , L = 0.024    , CALIB = 0.045831 / 0.024 / Brho , APERTYPE = CIRCLE, APERTURE = {0.060325} ;
SSEXT    : SEXTUPOLE     , L = 0.024    , CALIB = 0.045477 / 0.024 / Brho , APERTYPE = CIRCLE, APERTURE = {0.060325} ;
BPM_1    : MONITOR        , L = 0.024    , APERTYPE = CIRCLE, APERTURE = {0.060325} ;
```

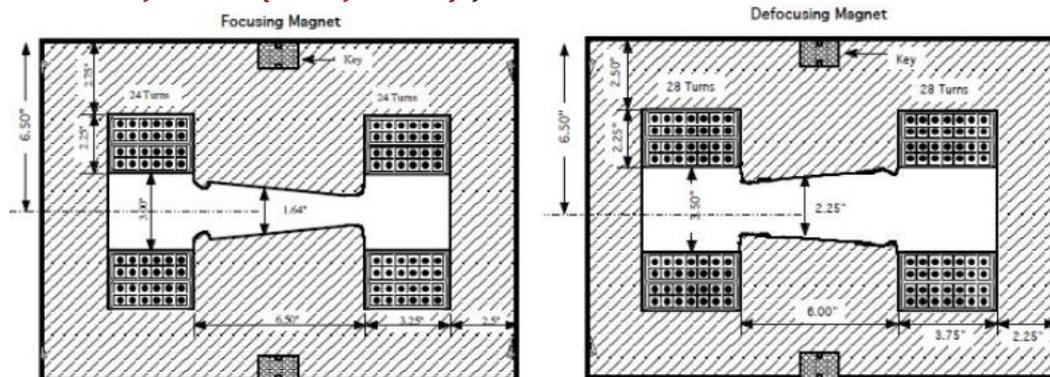
!----- Injection Orbit Bump Magnets -----

```
ORBUMPa_1 : VKICKER      , L = LORB_eff , KICK := ob_ang1+ob1_k0L_err, APERTYPE = RECTANGLE, APERTURE = {0.028,0.150} ;
ORBUMPa_2 : VKICKER      , L = LORB_eff , KICK := ob_ang2+ob2_k0L_err, APERTYPE = RECTANGLE, APERTURE = {0.028,0.150} ;
ORBUMPa_3 : VKICKER      , L = LORB_eff , KICK := ob_ang3+ob3_k0L_err, APERTYPE = RECTANGLE, APERTURE = {0.028,0.150} ;
ORBUMPa_4 : VKICKER      , L = LORB_eff , KICK := ob_ang4+ob4_k0L_err, APERTYPE = RECTANGLE, APERTURE = {0.028,0.150} ;
```

```
FMAG      : RBEND        , L = blengthf, ANGLE = blength/rhof, K1 = qsf , K2 = ssf, APERTYPE = RECTANGLE, APERTURE = {0.08255,0.020828} ;
DMAG      : RBEND        , L = blengthd, ANGLE = blength/rhod, K1 = qsd , K2 = ssd, APERTYPE = RECTANGLE, APERTURE = {0.07620,0.028576} ;
```

```
DMAGU11   : RBEND, L=blengthd-cut_length_d, ANGLE = DMAGU11_ang,K1 = qsd*(1/(1-fraction_d))*modd, K2 = ssd*(1/(1-fraction_d))*modd,
APERTYPE=RECTANGLE,APERTURE={0.0762,0.028576} ;
```

```
DMAGD11   : RBEND, L=blengthd-cut_length_d, ANGLE = DMAGD11_ang,K1 = qsd*(1/(1-fraction_d))*modd, K2 = ssd*(1/(1-fraction_d))*modd,
APERTYPE=RECTANGLE,APERTURE={0.0762,0.028576} ;
```



input beam parameters at foil

```
!! 800 MeV values:

ke      := 0.800 ;
etotal  := pmass + ke ;
momentum:= sqrt( etotal^2-pmass^2 );
beta    := momentum/etotal ;
value, etotal, momentum, beta ;

! values at foil:
bx0     := 6.2715157643 ;
ax0     := 0.0835620556 ;
by0     := 19.0101076906 ;
ay0     := 0.0367137761 ;
dx0     := 2.1182749319 ;
dpx0    := -0.0108615422 ;
dy0     := -0.0993769663 ;
dpy0    := -0.0001246919 ;
x0      := 0.0050359970 ;
px0     := 0.0000005332 ;
y0      := 0.1131905954 ;
py0     := 0.0000000000 ;

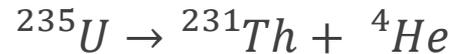
e95n    := 1.1541e-6 ;
e95     := e95n * pmass/momentum ;
value, e95 ;

sigmax  := sqrt( bx0*e95/6. ) ;
sigmay  := sqrt( by0*e95/6. ) ;
```

nuclear analog of H^0 decay equation

$$\frac{dH^0}{dt} = -\frac{1}{\tau} \cdot H^0$$

$$\frac{dH^+}{dt} = +\frac{1}{\tau} \cdot H^0$$



$$\frac{d({}^{235}\text{U})}{dt} = -\frac{1}{\tau} \cdot ({}^{235}\text{U})$$

$$\frac{d({}^{231}\text{Th})}{dt} = +\frac{1}{\tau} \cdot ({}^{235}\text{U})$$

$${}^{235}\text{U}(t) = e^{-t/\tau}$$

$${}^{231}\text{Th}(t) = (1 - e^{-t/\tau})$$

$$(\tau = 1E9 \text{ yrs})$$

another analog of H^- decay equation

$$\frac{dH^-}{d\omega} = -\sigma_{-0} \cdot H^-$$

- probability of decay is just proportional to # of particles present, with the proportionality constant the cross-section for stripping on a target atom.
- This can be converted to the probability of H^- stripping in time:
- $\omega = Nz$; $z = \text{thickness } (\mu\text{g}/\text{cm}^2)$; $N = \# \text{ atoms}/\mu\text{g}$ ($5E16$ for ^{12}C) ; and;
 $z = \rho x$; $\rho = \text{density } (\mu\text{g}/\text{cm}^3)$ ($2.25E6$ for ^{12}C) ; $x = \text{distance into foil (cm)}$

and with $x = \beta c \cdot t$, we arrive at (the not very useful) description of how the H^- strip in time as they traverse the foil:

$$\frac{dH^-}{dt} = -(\sigma_{-0} \cdot \beta c \rho N) \cdot H^- \equiv -\frac{1}{\tau} \cdot H^-$$

a tip for solving the species population equations

$$h^0(z) = e^{-\int_0^z dz' / \tau(z') \cdot \beta c}$$

$$H^+(z) = (1 - e^{-\int_0^z dz' / \tau(z') \cdot \beta c})$$

- solving these equations is not as bad as they might first appear because the exponential integrals can be somewhat simplified.

- define:

$$E(z) \equiv e^{-\int_0^z dz' / \tau(z') \cdot \beta c}$$

- then:

$$E(z_2) = e^{-\int_0^{z_2} dz' / \tau(z') \cdot \beta c} = e^{-\int_0^{z_1} dz' / \tau(z') \cdot \beta c} \cdot e^{-\int_{z_1}^{z_2} dz' / \tau(z') \cdot \beta c}$$

$$E(z_2) = E(z_1) \cdot e^{-\int_{z_1}^{z_2} dz' / \tau(z') \cdot \beta c}$$

- So, the problem reduces to accurate integration of the exponential term over the small range $\Delta z = z_2 - z_1$. As an example, even the accuracy of a 3-point Simpson integration is $O(\Delta z^5)$. Even smaller errors would result by, say, Runge-Kutta integration techniques.

Coraggio, Avanti !

Ω